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A MONTE CARLO RISK ANALYSIS OF LIFE CYCLE COST PREDICTION

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Wright-Patterson Air Force Base, Ohio

September 1975

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The model output is applied to the problem of determining appropriate contractor rewards or penalties, investigating the feasibility of contractor strategies, and investigating the effect of various test lengths.

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THESIS

GOR/SM/75D-6 Samuel B. Graves
Captain USAF

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A MONTE CARLO RISK ANALYSIS OF LIFE CYCLE COST PREDICTION

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

Samuel B. Graves, B.S.
Captain USAF

Graduate Operations Research

September 1975

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PREFACE

The analytical work presented here is an attempt to provide illumination in a field where decisions must be made concerning the expenditures of potentially large amounts of public money. These decisions have been made in the past on the basis of the intuition of knowledgeable and experienced people. It seems reasonable that these intuitive judgements may be made with greater precision if the analytical tools of operations research can be properly applied to these questions of efficiency in allocation of resources.

It is the author's hope that the kind of careful study presented here may lead to a more efficient, if not optimum, disbursement of the public monies. It is in this spirit that this work has been undertaken.

Gratitude is expressed to Captain Robert Tripp, who assisted in defining the study, Lieutenant Dwight Collins, who provided considerable insight into the analytical methods, and finally to my wife, Florence, who provided patient editorial assistance.

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ABSTRACT

This study is an investigation of the uncertainties involved in the prediction and measurement of Life Cycle Costs. The particular treatment here analyzes Logistic Support Costs, which are a subset of the Life Cycle Costs. The Logistics Supportability Incentives which are embodied in the current General Dynamics F-16 contract are analyzed in the light of the stochastic uncertainties of prediction and measurement of Logistic Support Cost.

A Monte Carlo Simulation model is developed which will approximate the uncertainties involved in obtaining a sample measurement of Logistic Support Cost in a fixed length test.

The model output is applied to the problems of determining appropriate contractor rewards or penalties, investigating the feasibility of contractor strategies, and investigating the effect of various test lengths.

Chapter 1

INTRODUCTION

Objective

The objective of this research is to investigate and provide insight into certain Department of Defense reliability and maintainability incentive contracting options. In particular, the options to be explored in detail are the correction of deficiencies (COD) and award fee provisions which are embodied in the current General Dynamics F-16 contract. The current provisions of the F-16 contract will provide a case study framework within which this enquiry may be conducted. The products of this study are intended to be useful not only to the managers of the F-16 acquisition, who must formulate plans for administration of their existing award fee and correction of deficiencies provisions; but also to managers who must plan future acquisitions which will employ reliability and maintainability incentives in the form of award fees and corrections of deficiencies clauses. The study will attempt to quantify and explain the considerable uncertainties involved in measurement of reliability and maintainability; this measurement being a logically necessary prerequisite to any rational exercise of the positive or negative incentive provisions of this form of contract.

Background

The Department of Defense has entered a new era of fiscal austerity. In order to maintain an effective force within the budget constraints of the future, the Air Force, along with the other departments, must conduct careful studies of the total cost impact of weapons system acquisitions. One of the tools for measurement of the total cost impact is the Life Cycle Cost concept. Under this concept the Air Force attempts to minimize the total Life Cycle Cost of a weapons system while maintaining a given effectiveness level. In general, the Life Cycle Cost of any system is the sum of the acquisition cost and the operating and support cost. In this thesis, in order to establish reasonable limits on the scope of investigation, it has been necessary to focus attention on a subset of the operating and support costs. This subset, the logistic support cost, will be described in detail in subsequent discussions. It is sufficient here to way that the logistic support costs comprise a significant portion of the overall operating and support costs. The decision-making activities discussed in this study will be those which are directly related to logistic support cost.

The F-16 contract has been structured with the goal of providing incentive to the contractor toward development and production of equipment which will demonstrate acceptable life cycle cost characteristics. To be specific, the F-16 contract has been written with both positive and

negative incentives to the contractor associated with the attainment of certain logistics supportability targets. The particular equipments to be covered under the logistics supportability incentives are called first line units (FLU's). A FLU is defined to be a first, second, or third level of assembly as described by MIL-M-38769-USAF. A FLU is the first level of assembly below system level that would be carried as a line item of supply at base level. A first line unit is roughly comparable to a line replaceable unit (LRU), but is a more precise definition which is more meaningful and useful for the purposes of measuring logistic support cost. In this context a FLU may be, for example, a line replaceable unit in the radar navigation system or a nose gear actuator in the landing gear system. There are two criteria for determining what equipments will be designated as FLU's. The first is that; within each system, fault isolation, removal, and replacement of FLU's will correct no less than 80 per cent of the failures in that system. The second criterion is that FLU's will generate at least 80 per cent of the required maintenance manhours on the system. The contractor then must rank his components within each system until the sum of their maintenance requirements will account for both 80 per cent of the failures and 80 per cent of the direct maintenance manhours. For the F-16 there are approximately 280 FLU's. These FLU's then, in theory at least, ought to account for at least 80 per cent of the maintenance effort in terms

of failures and manhours. These FLU's are a manageable and identifiable group of components which should account for a preponderance of the logistic support costs. To provide even greater support cost visibility, a subset of these 280 FLU's has been selected. The criterion for selection of this subset, designated the control FLU's, is that these control FLU's will contribute no less than 50 per cent of the total FLU level support costs. These have been informally called the "high burner" FLU's.

Different incentive arrangements have been made for the control and non-control FLU's. For non-control FLU's an award fee has been provided. This award fee in the amount of $6,400,000 may be paid fully or in part to the contractor at the government's discretion. If the measured logistic support cost of the non-control FLU's is less than the government established target logistic support cost of the non-control FLU's, then the contractor is eligible for the award. The measurement of the logistic support cost will be accomplished during a verification test, wherein the appropriate input parameters for an abbreviated version of the Air Force Logistics Command Logistic Support Cost Model (AFLC LSC model) will be determined. In the case of the non-control FLU's, no negative incentive is provided.

The provisions covering the "high burner" control FLU's are somewhat more complicated. The government has three separate incentive alternatives vis a vis the control FLU's.

The first alternative is the Support Cost Guarantee (SCG). This alternative provides a positive incentive in the form of an award fee, and a negative incentive in the form of a Correction of Deficiencies (COD) clause. The SCG provision is the primary object of investigation in this study. Briefly stated, the SCG alternative provides for an award fee of up to $2,000,000 in case the measured logistic support cost (MLSC) of the control FLU's as determined by the AFLC LSC model is less than the government determined target logistic support cost (TLSC). The measurement will take place in the above-mentioned verification test. The negative incentive, correction of deficiencies, is invoked in case the MLSC is greater than 1.25 times the TLSC. In this case the contractor must take action to correct the deficiency which caused the logistic support cost overrun. The costs of correction will be shared with the government on a 70/30 government/contractor sharing ratio in accordance with the provisions of the basic contract (1). The second alternative, the reliability improvement warranty (RIW), essentially provides for contractor maintenance of the control FLU's for a period of 48 months or 300,000 flying hours, whichever comes first. The third alternative, RIW with MTBF guarantee, provides not only for contractor maintenance but also for consignment (no charge) spares whenever the MTBF falls below the guaranteed level.

It is important to note that eac of the above

contracting alternatives may be applied on a FLU by FLU basis. The government will review the economics of each control FLU and select a contracting option for that FLU. If no FLU's are selected for reliability improvement warranties, then the total possible award fee under the COD provisions will remain at $2,000,000. However, for each FLU which is chosen for an RIW option there will be a proportionate reduction in the total amount of the possible award fee. This reduction will be in the same proportion as the logistic support cost of the chosen FLU is to the total logistic support cost. For example, if only one FLU, FLU X, was selected for a reliability improvement warranty, and if this FLU had a target logistic support cost of $5.0 million, then the possible award fee will be reduced by the fraction 5.0/TLSC. Given a projected TLSC of $38.4 million, then the total possible award fee based on the performance of the remaining control FLU's would be:

$$\$2.0 \text{ million} - \$2.0 \text{ million} \times (5.0/38.4)$$

or the total possible award fee would be $1.74 million. If all the control FLU's were selected for RIW, then of course there would be no award fee.[1]

There is only one prerequisite for contractor eligibility for some or all of the award fee. That

[1]A similar reduction in the award fee is made for each FLU which becomes Government Furnished Equipment (GFE).

prerequisite is, as mentioned above, that the measured logistic support cost be less than the target logistic support cost. This single prerequisite applies to both the control FLU award fee and the non-control FLU award fee. Since there are no further a priori conditions, the government has complete discretion in determining the amount of the fee, given that the prerequisite is met. It is one of the stated objectives of this study to provide some rational criteria for determining this amount.[2] Figure 1 is a decision logic chart describing the contracting alternatives and their effect on the award fee.

Structure of the Problem

As has been stated, the intent of the Air Force in this employment of the Life Cycle Cost concept, is to carefully measure the actual logistic support cost of the final product: or more precisely, to carefully measure a visible and manageable subset of the logistic support costs which should comprise a large percentage of the total. To this end an abbreviated logistic support cost model has been derived from the AFLC LSC model. According to this abbreviated model, the logistic support cost of a single control FLU can be represented as the sum of four inputs.

[2]It should be noted at this point that there are no engine module FLU's as the engine in this acquisition is GFE.

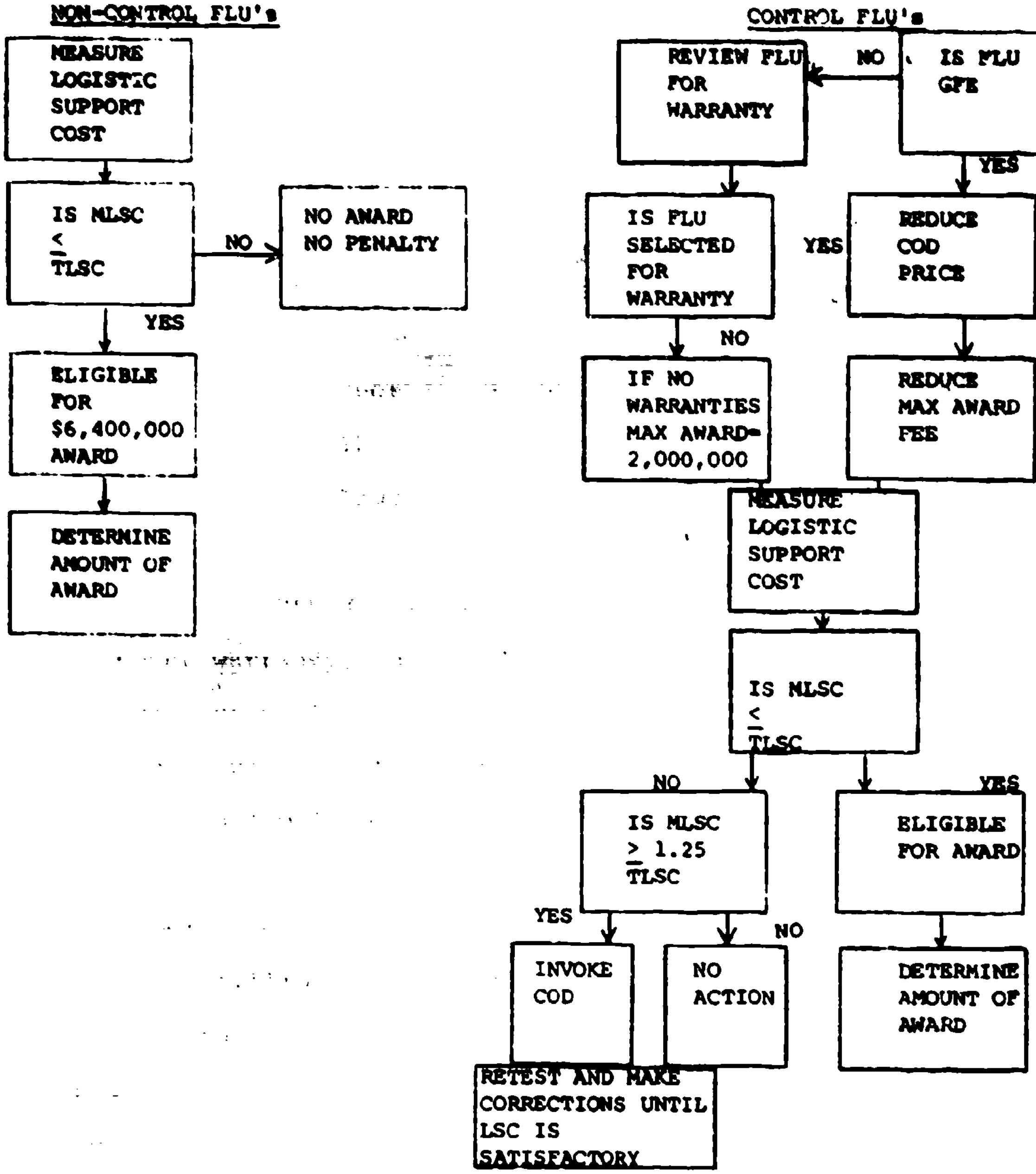

Figure 1. Contract Decision Logic Chart

These inputs are:

C_1, Initial and Replacement Spares

C_2, On Equipment Maintenance Costs

C_3, Off Equipment Maintenance Costs

C_5, Support Equipment Costs

It will be immediately evident to most observers that these four terms do not capture the totality of logistics support costs associated with each FLU. These four cost terms have been selected from the more comprehensive AFLC LSC model with the purpose of providing a visible, measurable set of costs over which the contractor should be able to exercise considerable control. The target logistic support cost then, which will be defined as the sum of C_1, C_2, C_3, and C_5 is not intended to represent all of the logistic support costs but rather to stand as a proxy for these costs. That is, the target logistic support cost is a representative standard against which a contractor's logistics performance may be measured. So, even though the model is not suitable as a tool for measuring total costs, it is a useful device for measuring contractor performance in cost control.

The total logistic support cost for the purposes of this study then is the sum over all the appropriate FLU's of each of these input costs.

That is:

Logistic Support Cost (non-control FLU's) =

$$\sum_{j=1}^{k} \{ C_{1j} + C_{2j} + C_{3j} + C_{5j} \}$$

where j represents the jth non-control FLU and k is the number of non-control FLU's.

Similarly:

Logistic Support Cost (control FLU's) =

$$\sum_{i=1}^{n} \{ C_{1i} + C_{2i} + C_{3i} + C_{5i} \}$$

where i represents the ith control FLU and n is the number of control FLU's.[3]

Each of the terms in the above equations is a function of numerous input parameters. The terms will be described in detail in subsequent chapters. Among the input parameters there are some which are deterministic and may be measured without error. An example of one such parameter is the unit cost of initial spares. At the time of the verification test this value will have been negotiated and will be a known, fixed quantity. Other input parameters, for example, mean time between failures, cannot be known with certainty. These parameters are the means of

[3]All further references to logistic support cost, unless otherwise specified, will apply to control FLU's only. As can be clearly seen by the reader, the methodology developed for the control FLU's is readily generalized to the other situations involving measurement or verification of supportability.

probability distributions which are estimated using the data collected in the verification test. Among all the input parameters to the LSC Model, there are only 13 which are actually subject to verification during the verification test. That is, we are interested in making awards, or invoking the correction of deficiencies clause only if these "subject to verification" parameters are the cause for a deviation from target cost. Clearly, no award or penalty action should be taken if a deviation from target cost occurs as a result of an exogenous variable such as inflation or a change in Air Force basing policy.[4]

Among the 13 parameters which are subject to verification there are three which will be susceptible to the greatest uncertainties in measurement. These parameters are:

1. Mean Time Between Failure (MTBF)
2. Maintenance Man Hour Expenditures (MMH)
3. Fraction of Failures Reparable This Station (RTS)

The measurements which are taken during the varification test will actually provide estimates for MTBF, MMH, and RTS. The reason that these are estimates and not true values is that the occurrences of Time to Failure, Time to Repair, and Reparable This Station are random variables.[5]

[4]Exogenous in the sense that it cannot be controlled by the contractor.

[5]A random variable is a numerical event whose value will vary in repeated samplings (42:56).

Since estimates of MTBF, MMH, and RTS are functions of these random variables, then they are themselves random variables. Since MLSC is a function of these estimates of MTBF, MMH, and RTS it too is a random variable.

Two hypotheses have been formed with regard to these three estimates.

1. The uncertainty in these estimates is in fact the major contributor to the uncertainty of the measured logistic support cost.

2. The contributions to uncertainty are, in descending order of magnitude, estimates of MTBF, MMH, and RTS.[6]

It is important to note that these hypotheses make no inference regarding the relative contributions of these factors to the total logistic support cost, but only make statements about how these factors impact on the uncertainty involved in measuring logistic support cost. It does not necessarily follow that those variables which have the greatest impact on total logistic support cost must also have the greatest impact on the uncertainty (variance) of the measured logistic support cost.

In order to develop a relationship between the variance of the logistic support cost and the input

[6]The uncertainty can be thought of as the amount of possible variation. It will be represented in this thesis by the mathematical variance which is defined as follows: The variance of a set of observed measurements, $Y_1,\ldots,Y_n$ is the average of the square of the deviation of the measurements about their mean. Symbolically: $V(Y)=\sum_i(\bar{Y}-Y_i)^2/(n-1)$.

parameters which are related to randomly varying phenomena, it is necessary to hypothesize some probability distribution for each of these random variables. Alternatively an attempt could be made to fabricate an empirical distribution; however, in this study, partially as a result of the paucity of historical data, the approach has been to use hypothetical probability distributions which are well justified by both theory and empirical evidence where available. When the probability distributions of the input parameter estimates are determined, then the problem is to relate the variance of the measured logistic support cost to the variance of these estimates. This may be done analytically if possible, or if not, then it may be accomplished by computer simulation. In this thesis the relationship has been developed through computer simulation due to analytical difficulties which will be explained in a subsequent chapter. Figure 2 illustrates the relationship between input uncertainty and MLSC uncertainty.

Realizing that the verification test will be run for only a total of 3500 hours, it is apparent that we will be dealing with small sample sizes. For example, one FLU which has a predicted MTBF of 563 hours would on the average experience about six failures in 3500 hours. It is not inconceivable that an item with a true MTBF of 563 hours might survive for 3500 hours with only one or two failures. Based on this very small sample of failures it

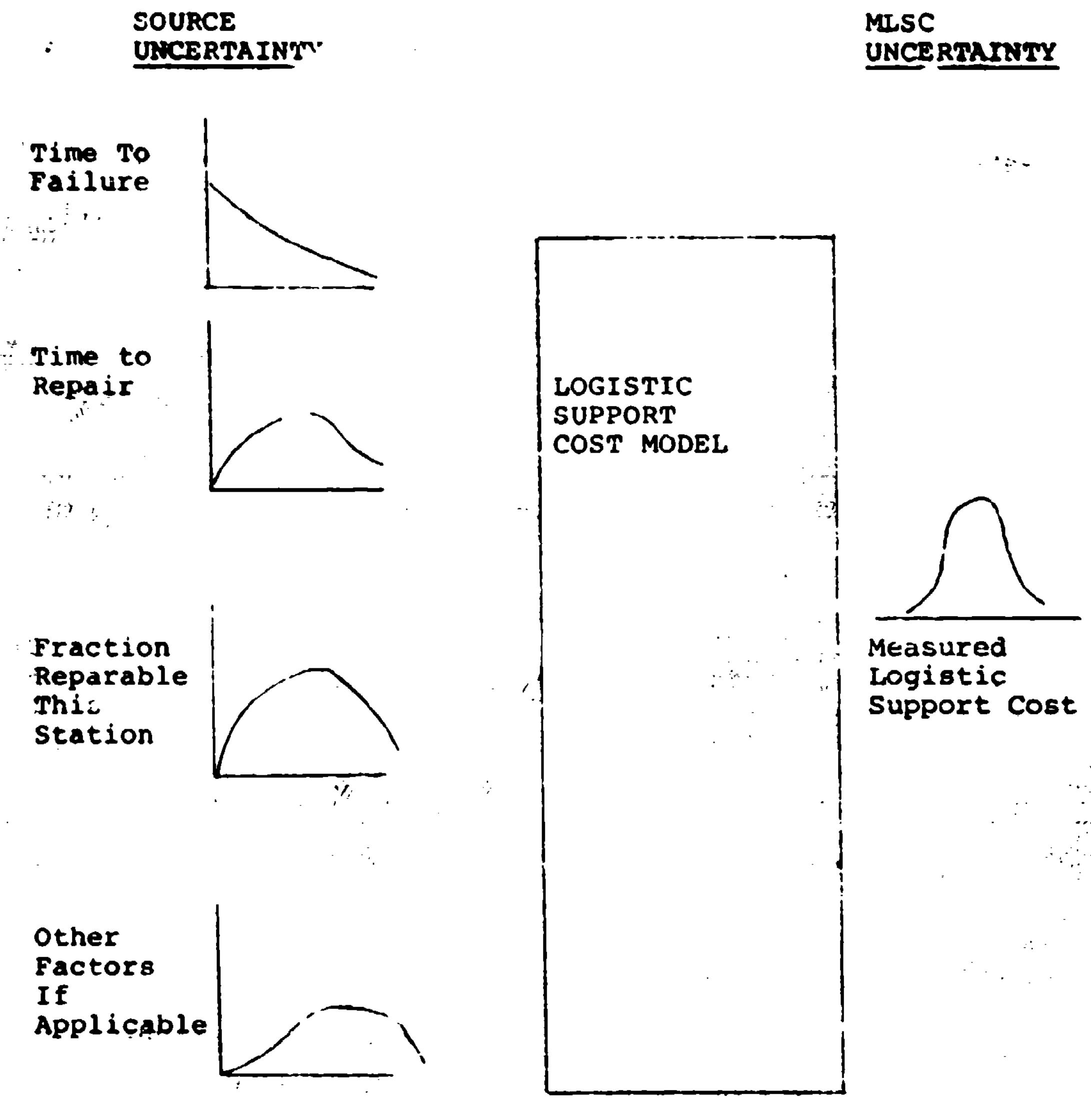

Figure 2. Uncertainty Sources

is necessary to make a prediction of the 15 year life cycle cost of the equipment. Consider again the FLU with the 563 hour true MTBF. As stated this FLU will have on the average six failures in 3500 hours. A 95 per cent confidence interval for the mean time between failures for this FLU would then be 563± 459; a very wide confidence interval.[7] It can be seen then that the 3500 hour test is in essence a single observation of the random variable, MLSC, which is based on small samples of input random variables. It would not be surprising then to find that the MLSC has a relatively large variance.

Once the variance and the type of probability distribution for the MLSC have been determined, then it is possible to make statistical inferences based on the results of a single 3500 hour test. For example, as will be shown, it is possible, given a single sample observation of MLSC, to say with what level of confidence that particular observation implies either a real target cost underrun or a real logistics support cost in excess of 1.25 times target cost. That is to say, we can derive information which will be useful in determining the probability of a correct decision.

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[7] Using the approximation: $\sigma_{\substack{\text{estimator}\\ \text{of mean}}} = \sigma/\sqrt{n}$ where n = number of failures

and $\sigma_{\text{exponential}} = \text{mean}_{\text{exponential}}$

so $\sigma_{\substack{\text{exponential}\\ \text{MTBF}}} = \dfrac{\text{MTBF}}{\sqrt{n}}$ and $\sigma_{\substack{\text{exponential}\\ \text{MTBF}}} \approx \dfrac{\text{Estimate of MTBF}}{\sqrt{n}}$

By varying the length of the test above and below 3500 hours in the simulation model some additional information can be derived. For example, we can determine how much an increase of 1000 hours of test time will increase the probability of making a correct decision.

Using these concepts, a series of decision curves can be constructed. These curves will show for a given test length how much confidence can be placed in any single observation of MLSC.

In view of the embarrassment to the Air Force which would result from the decision to present an award fee to a contractor whose product subsequently showed excessive support costs, it is clear that this decision ought to be made with the benefit of some statistical analysis. Conversely, the Air Force certainly would not want to invoke a correction of deficiencies without a reasonable confidence that a support cost overrun has occurred. Indeed, since the Air Force must pay 70 per cent of the costs of correction of deficiencies, an erroneous invocation of COD would unfairly penalize not only the contractor but also the government. Since the COD provisions are thoroughly specified in the F-16 contract, this study can provide statistical knowledge of the likelihood that a given decision to invoke COD or make an award was correct. Further, as will be subsequently shown, the methods developed here can provide substantial insight to those who must develop the correction of deficiencies

provisions for future acquisitions.

While conducting an investigation of the uncertainties involved in measurement of logistic support cost, some useful by-products are obtained. Using the mathematical model developed in this study it is possible to conduct numerous sensitivity analyses. Some examples are: sensitivity of the variance of MLSC to the number of FLU's in the test and sensitivity of LSC to stock level policy. These two analyses are shown in Appendices A and C respectively.

Since it appears that the Life Cycle Cost concept is here to stay, and inasmuch as the methodology to be described here may be readily generalized, it is apparent that these techniques ought to be of significant value to the Department of Defense.

Nature and Sources of Data

The primary source of data for the F-16 implementation of this study is the F-16 acquisition contract. Hence, the source of most of the data is the contractor. A brief discussion of the strengths and weaknesses of this data follows.

Consider first the contractor furnished MTBF predictions for the control FLU's. Most of these predictions are based at least in part on similarity to some existing Air Force equipment on either the A7D or the F-111. Field MTBF data on this baseline equipment has then been modified

by a complexity factor. For example, if a particular piece of equipment has 30 per cent fewer components than the baseline FLU, the complexity factor might be something like 1.4. That is, the F-16 FLU would be credited with an MTBF of about 1.4 times the baseline FLU. This figure would be further modified by the application of a usage and environmental factor, which attempts to reflect the impact of anticipated environmental stresses on the MTBF.

Maintenance manhour figures used were based on similar aircraft data, modified by complexity and size relationship factors established by the General Dynamics support requirements division. The G.D. Base shop simulator model was used in this analysis (1).

An assessment of the validity of these predictions is a highly subjective exercise. Experts in the reliability and maintainability field have stated the following with regard to predictions based on historical data.

> If the same organization, employing the same personnel has demonstrated an effective and extensive quality control organization...it is probably safe to assume that the reliability inherent in the new system will not be degraded as the design is translated from drawing to hardware any more than was the case in earlier programs [35].

If we accept this assertion, and if the stated preconditions are met, it would seem reasonable to compare the field performance of F-111 equipment to the predictions made by General Dynamics. A review of the F-111 category II test report indicates that for the eight avionics equipments for which MTBF predictions were provided, the measured MTBF

was on the average equal to one-third of the prediction (51). Considering this, and the history of MTBF predictions, where according to experts in the field, it is not uncommon to see an order of magnitude reduction in MTBF from laboratory to field conditions, it seems prudent to investigate the behavior of the LSC model with MTBF's degraded from those predicted by the contractor. This has been accomplished as a part of the analysis (52:27).

Projecting realistic maintainability estimates for tactical fighter aircraft during conceptual and development design phases is a continuing problem for the Department of Defense. Figure 3 gives a comparison of predicted to actual maintenance manhours per flight hour (52:18).

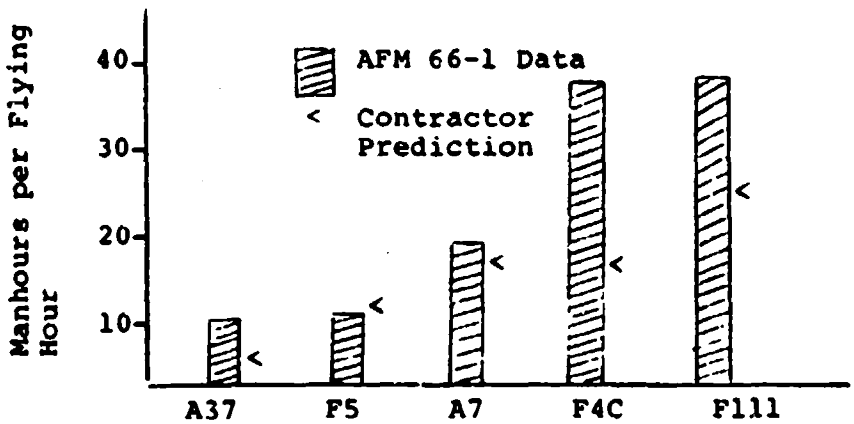

Figure 3. Manhours per Flying Hour Forecasts

It is clear that overly optimistic estimates are prevalent in maintainability as well as in reliability. On the average for the samples above, contractor estimates have been about 70 per cent of actual required. The General Dynamics

estimate for the F-111 was approximately 65 per cent of the actual field MMH/FH requirements.

There are clearly uncertainties regarding the accuracy of predicted reliability and maintainability levels. The purpose of the verification test is to determine whether the contractor has come acceptably close to the predicted values.

Summary of the Methodology

The first, and in a sense, the most important step in this analysis is the determination of the important random variables in the cost equations. That is, among the input parameters which are subject to verification, which are significant in terms of their impact on MLSC uncertainty? And, among those parameters, which can be considered deterministic constants as opposed to random variables? So, to be of interest in this methodology, the parameter must pass two tests.

1. Is the parameter subject to verification?
2. Is the parameter the mean of a probability distribution of a random variable?

The LSC input parameters which are subject to verification are listed below.

1. N: number of FLUs
2. QPA: Quantity of like FLUs within the system.
3. K: number of line items of peculiar support equipment (AGE) for ith FLU.

4. MTBF: Mean time between failures of the FLU in operating hours, in the operational environment.

5. RIP: Fraction of FLU failures reparable in place.

6. RTS: Fraction of removals reparable at base level.

7. NRTS: 1-RTS.[8]

8. COND: Fraction of removals resulting in condemnation at base level.

9. All Maintenance manhour data: (IMH,PAMH,BMH,DMH).

10. DOWN: Percentage of down time for the jth piece of peculiar age.

11. UC: Expected unit cost of the FLU at initial provisioning.

12: CAB: Cost per unit of support equipment at base level.

13. CAD: Cost per unit of support equipment at depot.

It is immediately apparent that N, QPA, and K are known constants. It has been previously stated that the value of UC will be a known quantity at the time of the verification test. Contractor data indicates that the value of K is zero, i.e., there are no items of peculiar AGE for any control FLU. Since K=0, we can eliminate DOWN, CAB, and CAD, all of which apply to peculiar AGE.

[8] In general it would not be correct to say that NRTS=1-RTS because; of that fraction of failures which is not reparable this station, some portion may be condemned at base level. So that, in general, NRTS=1-RTS-COND. In this application COND=0 for all control FLUs.

This leaves as candidates for inputs MTBF, RIP, RTS, COND, PAMH, RMH, BMH, and DMH.

COND is zero for all control FLUs so it is eliminated. RIP is zero for all but one control FLU, the flight control computer, for which the value is .01. Since this input would have an insignificant impact on the overall uncertainty, it is treated as a deterministic variable. Since the value of RIP is near zero, the aggregate value of IMH (in place repair manhours) will also have an insignificant impact on the overall uncertainty and thus it can be approximated by a deterministic variable.

Those parameters which will be used in connection with random variables then are: MTBF, RTS, PAMH, BMH, RMH, and DMH.[9]

Having identified these parameters, the next step in this, or any similar analysis, is to determine the probability distribution functions (PDF's) of the random variables to which these parameters are related. There are essentially two approaches to this task; the theoretical approach, and the empirical approach.

In the empirical approach, the preferred procedure would be to collect reliability and maintainability data on each of the FLUs and construct empirical distribution

[9] NRTS must be treated in the same manner but it will not be explicitly represented since NRTS=1-RTS.

functions using this data. It is very likely though that this information may be unavailable or nonexistent, particularly for new systems. The data collected by the Air Force manual 66-1 system is aggregate data which is useless for purposes of constructing histograms of time to failure or time to repair. An alternative approach which makes use of some historical data is the use of category II test and evaluation information from similar aircraft systems. These test and evaluation reports, published by the Air Force Flight Test Center, contain empirical probability distributions for time to failure and time to repair on the important subsystems of the aircraft. From this information some reasonable inferences can be made about new systems if the systems are similar (49; 50; 51).

In a purely theoretical approach it would be appropriate to simply hypothesize a distribution which, according to theory, should represent the process in question. Considerable discussion of this question is available in the literature of reliability and maintainability.

This analysis has drawn on information from both of these sources. In the initial stages of the analysis, PDF's were chosen for time to failure and time to repair based on theory. As historical information became available in the form of test reports for the A7D and the F-111, it largely confirmed the theoretical distributions. When there was reasonable doubt as to the correct distribution, sensitivity analysis was conducted to determine what would be the impact

of an error in choice of distributions. The same general approach has been used to determine appropriate distributions for both MTBF's and maintenance manhours. There is, of course, a myriad of diverse tasks associated with each FLU failure, but it is not the purpose of this undertaking to study each of these maintenance activities in detail. Rather, the approach has been to assume that all maintenance activities can be fairly represented by a single PDF.

A final assumption was made regarding time to failure and time to repair. It has been assumed here that the mean of each distribution will be constant over the period of the test. This means no allowance will be made for MTBF growth or a personnel learning curve during the six month period of the test. Information from the inertial navigation contractor is somewhat at variance with this assumption (57). This contractor has planned for considerable MTBF growth during the initial operational period. The planned growth rate if assumed to be linear would indicate a change of 17 per cent in the INS MTBF during the six months of the verification test. The assumption here is that this discovery does not do great violence to the findings of this report.

In order to formulate a probability distribution for RTS it is only necessary to visualize this process as a Bernoulli trial such as the toss of a biased coin. This is essentially the process which occurs with each FLU failure. With this in mind there is no alternative other than the binomial distribution.

With all of the above decisions made, the next step in the analysis is to estimate the expected value and variance (E (MLSC) and V (MLSC)) of the measured logistic support cost as a function of the input parameters MTBF, MMH, and RTS.

That is:

E(MLSC = f(MTBF, MMH, RTS)

AND

V(MLSC = F(MTBF, MMH, RTS)

The difficulties associated with an analytical approach to the above problem will depend on the underlying probability distributions. In this particular application of the methodology the stumbling block to analytical expression is in the stock level equations which will be covered in detail in Chapter 4. For this application, Monte Carlo Simulation has been used in this part of the analysis.

Although the simulation model will be covered in detail in Chapter 4, a brief summary description is presented here. The simulation model essentially duplicates the transactions and conditions of the 3500 hour verification test. The model first generates random failures for each control FLU over a 3500 hour period, based on either an exponential or a Weibull distribution of failures as selected by user. For each failure of the ith FLU, the model then samples from the binomial distribution to determine whether the failure will be reparable at this station or not reparable this station. For each failure, the simulation samples from a manhour generator to determine

preparation and access manhours (PAMH), and removal manhours (RMH). This sample is taken from either a lognormal or a Weibull distribution as chosen by the user. No stochastic sample is taken for in place manhours (IMH) as this is treated as a deterministic variable. Next, for those failures which have been determined to be reparable at base level, the model generates a sample for base manhours (BMH) from the appropriate manhour generator. For those failures which were not reparable at base, the model generates a sample for manhours to repair at depot (DMH).

At the conclusion of each 3500 hour test, the model calculates an estimate for MTBF, PAMH, RMH, BMH, DMH, and RTS. Given these mean values the model then calculates an appropriate stock level for initial spares for the ith FLU based on a Poisson demand rate and a government established expected backorder value (EBO).

Finally, all of this information is input to the AFLC abbreviated LSC model which outputs one sample point of MLSC.

Since it is certain that not all readers will agree as to the propriety of the stochastic inputs to the simulation, it is necessary to conduct an analysis to determine the sensitivity of the model to the form of the inputs. All possible combinations of the inputs where each is allowed to be deterministic or stochastic produced eight data points. If the verification test length is treated as an input, then there are 16 data points. Standard analysis of variance techniques have been employed in Chapter 4 to examine the

effects and interactions of each input variable. In addition, a separate analysis was conducted to determine the sensitivity of V(MLSC) to the type of PDF employed for failure and manhour generators. These analyses will illuminate not only the sensitivity of the model to the assumptions, but also the contribution of each of the input random variables to the overall uncertainty of the output. This information is useful in determining the relative importance of careful measurement for each of the input random variables.

Determination of the form of the MLSC distribution is accomplished by goodness of fit tests. The Kolmogorov-Smirnov test has been employed and the results are shown in Chapter 4.

After the form of the distribution is determined, then statistical inferences can be made based on a single observation of MLSC and a "decision curve" constructed.

The final step in the implementation of the methodology is the construction of an award fee curve. This curve is constructed with the benefit of knowledge derived from the decision curves and with the provision for subjective inputs from performance evaluation.

As a result of this investigation, it has been possible to:

1. Describe the major factors in the uncertainty of life cycle cost estimates and their relative importance as embodied in the F-16 contract.

2. Describe the probability distribution function of the measured logistics support cost for various input distributions and assumptions.

3. Determine the effect of changes in verification test length on the uncertainty of the measured logistic support cost.

4. Provide planning factors for managers who will formulate future reliability incentive contracts with regard to test length, number of items in the test and output uncertainties.

In the following chapter, a brief discussion is presented of the prior research which has been conducted in this area by the RAND Corporation and ARINC Research Corporation.

Chapter 3 discusses in greater detail the contractual provisions which are important in this study. The AFLC logistics support cost model is explained along with the rationale for the abbreviated model. A brief discussion of relevant versus irrelevant costs is included in support of the abbreviated LSC model.

In Chapter 4 the simulation model employed is described in sufficient detail so as to be understood by those who are not acquainted with the field of simulation. The technical details of the simulation model along with the actual computer program are included in Appendix D. A description of the goodness of fit tests performed on the distribution of MLSC's is similarly found in Chapter 4

with the technical details in Appendix D.

A thorough analysis of the contributions of each uncertain input parameter to the overall uncertainty is included in Chapter 4. The vehicle for the analysis is an experimental design based on four inputs: MTBF, MMH, RTS, and test length.

Chapter 5 demonstrates the practical applications of the simulation methodology. Included are three applications:

1. Determination of a suitable COD invocation ratio and test length.

2. Analysis of a contractor strategy.

3. Development of award fee design.

Finally, Chapter 6 includes a summary and generalization of the methodology, conclusions, and recommendations for further study. A thorough outline is presented for a proposed study of optimum verification test length.

Chapter 2

PRIOR RESEARCH INTO LIFE CYCLE COST PREDICTION AND CORRECTION OF DEFICIENCIES PROVISIONS

Introduction

In this chapter, a brief discussion of the important prior research in this area is presented. The purpose of the discussion is to compare the assumptions of the different approaches and demonstrate the consistency in results. This discussion should serve to unify the various independent studies which have been conducted in this field to date.

Uncertainty in Life Cycle Cost

The primary source of prior research in this area of study is a working paper by the Rand Corporation (62). The Rand study was directed toward the problems of confidence in Life Cycle Cost estimates and the practicality of Life Cycle Cost models as aids to acquisition decision making.

In this discussion the author, E.S. Timson, compares the uncertainty of Life Cycle Cost estimations at various times in a program development cycle. He concludes that the uncertainties in Life Cycle Cost prediction during the development phase are so great as to make the AFLC Life Cycle Cost Model of questionable utility in supporting policy decisions such as enforcement of warranties or establishment of operating cost targets.

The paper conducts a case study of the N-16 inertial measurement set logistic support costs. In this particular example, there is an extreme disparity between development phase prediction of LSC and that which was measured during the operational test and evaluation (OT&E) phase. The predicted LSC was $14.727 million. The measured LSC was $131.565 million.

Timson points out in this analysis that there are three basic methods for establishing the uncertainty (variance) of the measured logistic support cost as a function of the uncertainty (variance) of the input random variables. These three methods are convolution integrals, Monte Carlo methods, and theory of errors. He continues to state that convolution integrals are difficult to use unless there is a relatively simple relationship between the input and output distribution. Clearly in the Life Cycle Cost Model this relationship is not simple. Monte Carlo methods, as he says, can accommodate any functional relationship between input and output, and can even make provisions for dependent inputs. Error theory, which Timson uses is adequate for a relatively simple functional relationship where the input random variables have normal distributions. The error theory approach to this problem uses the following mathematical relationship.

The variance of some output V(Y) which is a function of n independent inputs $X_1 \ldots X_n$, is given by

$$V(Y) = (\frac{\partial Y}{\partial X_1})^2 V(X_1) + \ldots + (\frac{\partial Y}{\partial X_n})^2 V(X_n)$$

As will be shown in Chapter 4, the error theory approach breaks down in the F-16 application due to the mathematically intractable stock level calculation equations. Some question arises also as to the propriety of the normality assumption, particularly with regard to the distribution of the random variable RTS which is sampled from the binomial. Neither of the two conditions for normality are met, i.e., that RTS be close to .5 and the number of samples close to 30.

An analysis of the input variables used in Timson's analysis is very interesting. For example, for the input distribution of MTBF, a normal distribution is specified with a standard deviation equal to 10 per cent of the mean. No information is available with which to make a meaningful estimate of such a distribution of MTBF's for this (F-16) application, but this problem is avoided by using the actual distribution of failures which is known with considerable confidence. It is worthwhile however, to attempt to approximate Timson's result using F-16 data to determine whether there is at least order of magnitude agreement between the two different approaches to the problem. The Timson study is based on a FLU which is demonstrating a 40 hour MTBF during OT&E. Using the relationship $\sigma_{MTBF} = \frac{MTBF}{\sqrt{n}}$ where n = number of failures, an F-16 FLU with a 40 hour true MTBF would experience approximately 88 failures during a 3500 hour test. The standard deviation for the mean time

between failures then would be

$$\sigma_{MTBF} = \frac{40}{\sqrt{88}} \quad \text{or} \quad \sigma_{MTBF} = 4.26$$

This result is in remarkably close agreement with Timson's approximation. Of course, without knowlegde of the length of the OT&E he is describing, it is not possible to determine whether this apparent consistency is real or coincidental.

It is also possible to compare the distribution of RTS used in the current study with Timson's application. Once again, treating the event reparable this station, not reparable this station, as a Bernoulli trial the events would be distributed with mean n x RTS and variance n x RTS x (1-RTS). (n is the number of failures)[1] using n=20 and RTS=.95 we have:

Mean = 20 x .95 = 19

Variance = 20 x .95 x .05 = .95

Standard Deviation = .975

So the standard deviation here is 5.13% of the mean. Timson has used 10 per cent of the mean for the standard deviation of his normal distribution of RTS.

Overall, it would appear that there is reasonable consistency between inputs in the simulation approach and Timson's approach. Some consistency can also be seen in

[1]Using here the normality assumption which as stated is questionable in this application.

the output observations. For example, Timson estimates the standard deviation of the LSC estimate during OT&E to be 19 per cent of the mean. The current analysis, with all three random variables included outputs a standard deviation of 10.3 per cent of the mean. Driving the variance of MTBF to zero reduces the output standard deviation of the Timson model to 12 per cent of the mean. A similar change in this model shows an output standard deviation of 7.0 per cent of the mean. Finally, by making all variables except RTS deterministic, the Timson model shows a standard deviation of 5 per cent of mean while this model shows 7.0 per cent. Table I below summarizes these findings,

TABLE I

ANALYSIS OF STANDARD DEVIATION TO MEAN RATIOS

	THIS STUDY (13 FLUs) (Per cent of mean)	RAND STUDY (1 FLU) (Per cent of mean)
All inputs stochastic	10%	19%
$\sigma_{MTBF=0}$	7%	12%
$\sigma_{MTBF=0}$ $\sigma_{MMH} = 0$	7%	5%

It is apparent that Timson's estimates are larger than those in this study except when $\sigma\binom{MTBF=0}{MMH=0}$, where they are approximately equal. This is not surprising in view of two facts:

1. Timson's study addresses only one FLU which results

in a larger overall variance to mean ratio.[2] 2. Timson's study includes several random variables not included in this study. (Six random variables are included in C_5 which is zero in this application.)

To test further for consistency between the two approaches, the model in this study was run for only one FLU with all three factors stochastically input. The result was a standard deviation of 29 per cent of the mean which is in reasonably close agreement with the 19 per cent found in the Rand study.

The Rand study concludes with the comment that the deterministic use of the accounting type of logistics support cost model does not seem appropriate. It further suggests that a stochastic simulation model would be one possible solution to meaningful use of the AFLC LSC model. This suggestion is the genesis of the current study.

Analysis of COD Provisions

An exhaustive study of the development and analysis of RIW and COD provisions for the air combat fighter was conducted by the ARINC Research Corporation in early 1975 (17). The purpose of the study was to analyze the Life Cycle Cost controls and warranty provisions as applied to the Air Combat Fighter (ACF), to include suggested improvements

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[2] See Appendix A for an analysis of the change in the mean to variance ratio of MLSC as a function of the number of FLU's tested.

in the contract provisions as well as an evaluation of contractor responses. As such, the ARINC study provides a stepping off point for this analysis and deserves a brief description.

Of particular interest is the ARINC analysis of the correction of deficiencies option. At the point in time when the ARINC study was conducted, General Dynamics had made a proposal regarding a possible reduction of their COD bid price. The initial COD bid requirement was based on a correction of deficiencies if MLSC was greater than TLSC. General Dynamics offered to reduce their COD bid price by $4,005,000 if the Air Force would agree not to impose a COD until MLSC exceeded TLSC by more than 15 per cent. By sensitivity analysis on the Air Force LSC model, ARINC determined that a reduction of average MTBF to 85 to 90 per cent of the specified value would result in an MLSC equal to 1.15 times TLSC. Further analysis showed that a 25 per cent increase in TLSC would result from a reduction in MTBF to about 77 to 80 per cent of specified.

ARINC analyzed the uncertainty in the MLSC measurement in the following manner. Consider a FLU with a true MTBF of 175 hours. During a 3500 hour test, about 20 failures would be expected. If the achieved MTBF were only 77 per cent of specified, about 26 failures would be expected. ARINC then reasoned that because of the uncertainties involved in the verification test measurement methods, and the fact that the causes of some failures are certain

to be questionable, there would not be a statistically significant difference between 20 and 26 failures at a reasonable confidence level. Indeed, after consideration of the Rand study which estimated a 40 per cent "false pull" rate, it is apparent the results of the verification test could be swayed in either direction by the weight of "false pulls" (52:4).[3] The estimate of a 40 per cent false pull rate is confirmed by Balaban (5:20).

Using the methods demonstrated in Chapter 1, it is possible to develop a confidence interval about an MTBF derived from a given test length. Using the above example and assuming an exponential failure rate:

$$\underset{(n=20)}{\sigma_{MTBF}} = \frac{175}{\sqrt{20}} = 39$$

$$\underset{(n=26)}{\sigma_{MTBF}} = \frac{134}{\sqrt{26}} = 26$$

So, a 70 per cent confidence interval for MTBF would be:

n=20 (136-214)

n=26 (108-160)

Since the confidence intervals overlap at a relatively low confidence level of 70 per cent, it seems clear that a difference between 20 and 26 failures would not be a sufficiently reliable distinction. The conclusion which

[3] A false pull is defined here as a maintenance removal which was subsequently checked and found to be operating correctly.

holds here for only a single FLU with one source of uncertainty is generalized to many FLUs and several sources of uncertainty in the current analysis.

ARINC recommended the following: If General Dynamics would continue to reduce their COD price at the rate offered ($4.0 million for a 15 per cent no fault zone, or $267,000 for each no fault percentage above TLSC), the Air Force should agree to a 25 per cent no fault zone above the TLSC and a reduction in the original COD price. This was accomplished. In a further analysis of this particular issue this study will show how a given amount of certainty can be provided to the contractor by increasing test length rather than necessarily relaxing the constraint upon the contractor as was done in the F-16 contract. For example, it can be shown that under the original F-16 COD provision, wherein the COD would be invoked if MLSC was greater than TLSC, the contractor is exposed to a 50 per cent chance of an incorrect invocation of COD. Under the provision that COD would not be invoked until MLSC was greater than 1.25 times COD, the contractor's risk of erroneous invocation of COD is only about 2 per cent. If the amount of risk which the contractor is prepared to take can be established in this manner, then the Air Force can provide that risk level to the contractor in two ways. The first is as above, adjustment of the invocation point. The second is adjustment of test length. The actual methodology for the determination of test length is given in Chapter 5.

The ARINC study reviewed the experience of the Air Force in RIW/COD contracting and, among their recommendations was this: "Establish and specify the criteria to be used for determining the amount of the award fee that the contractor will receive if the MLSC is equal to or less than TLSC." This recommendation provides one of the stimuli for the current study.

Chapter 3

CONTRACT PROVISIONS AND THE LOGISTIC SUPPORT COST MODEL

Introduction

In this chapter the important contractual provisions which are critical to this analysis are discussed in sufficient detail to provide a thorough understanding of the correction of deficiencies clauses and the procedures for the verification test.

A summary of the AFLC logistics support cost model is also presented. Each term is explained along with a discussion of the reasoning and assumptions involved in including or excluding it from the abbreviated logistic support cost model employed for a FLU level analysis.

Contractual Provisions for Correction of Deficiencies and Supportability Verification Test

There is a total of $8,400,000 in award fees potentially available to the contractor. As previously stated, this is divided into two separate award fees of $2,000,000 and $6,400,000.

The $2,000,000 fee is based on the logistic supportability of the control FLUs. There will be either 12 or 13 control FLUs depending on which radar contractor is ultimately chosen. The control FLUs for both potential

radar suppliers are shown in Table II.

It can be seen from Table II that there will be 13 FLU's if Hughes radar is purchased, or 12 FLU's if Westinghouse radar is purchased. In order to provide a vehicle for the methodology without duplicating every step, the Hughes radar was chosen for this analysis based on discussion with AFLC AQMLA (2). The function of the model and methodology is identical for either radar and the reader can readily reconstruct the methodology for the Westinghouse radar if desired.

The contract provides for changes to the target logistic support cost of control FLUs (TLSC-COD) and the target logistic support cost of non control FLUs (TLSC-System) in six specific cases. There are

1. Approved Engineering Change Proposals (ECP's) in conjunction with individual renegotiated values resulting from the engineering change.

2. Changes in the anticipated force structure of activity levels to be supported.

3. Inflation factor adjustments to acquisition cost elements.

4. Changes to factors defining the maintenance concept resulting from a government approved repair level analysis.

5. Adjustment due to selection of radar.

6. Adjustment due to subsequent identification of certain control FLUs designated as government furnished

TABLE II

LIST OF FLU's

FLU	WORK UNIT CODE (WUC)
Headup Display	74 BAO
Navigation Unit	74 DAO
Fire Control Computer	74 CAO
Electronics HUD	74 BCO
Flight Control Computer	14 ABO
Radar EO Display	74 EAO
Digital Scan Converter	74 ECO
Electronics EO Display	74 EBO
WESTINGHOUSE RADAR	
Antenna Servo	74 AAO
Low Power RF	74 ABO
Digital Processor	74 AEO
Transmitter	74 ACO
HUGHES RADAR	
Receiver Exciter	74 ABO
Data Processor	74 AEO
Signal Processor	74 ADO
Transmitter	74 ACO
Antenna	74 AAO

Note: Detailed Specifications for each FLU are found in Appendix B.

equipment (GFE).

The above are the only allowable adjustments to TLSC. It is important to note then that inflation is essentially a "flow through," or an exogenous variable. The provision which allows for adjustments due to changes in activity level is necessary because, as will be shown, the LSC equations are driven by the terms peak force flying hours (PFFH) and total force flying hours (TFFH). It will become apparent in the analysis that a change in TFFH or PFFH will not affect the overall probability distribution of MLSC's. This observation is confirmed by the ARINC study (17:A-13).

The contract clearly states that the values of MTBF will not be renegotiated, and further, that any changes to organizational, intermediate, or depot level man hour values shall retain the same gross weighted manhour cost value (man hours expended times labor rate). Symbolically, this could be stated:

$$\text{If } (PAMH \times BLR + IMH \times BLR + BMH \times BLR + DMH \times DLR) = K$$

$$\text{Then } (PAMH_1 \times BLR + IMH_1 \times BLR + BMH_1 \times BLR + DMH_1 \times DLR) = K$$

BLR: Base labor rate
DLR: Depot labor rate

In other words, the workload can be shifted among the organizational levels, but no negotiated change in the aggregate labor cost per FLU is allowed.

The Verification Test

The verification test to collect data for the purpose

of measuring the contractor's eligibility to receive the award fees will be conducted by the Air Force using the first operational combat crew training squadron (CCTS). The test will begin six months after activation of that squadron. It is assumed then, that by the sixth month after activation, the problems of infant mortalities in avionics equipment should be over. This assumption is necessary to the use of the exponential distribution for generation of failures. If a test were to be conducted during a period when infant mortalities were being experiences it is likely that a much higher than specified failure rate would be observed. It would be necessary to use a Weibull distribution of failures with a shape parameter m<1 to represent such a process. See Figure 4.

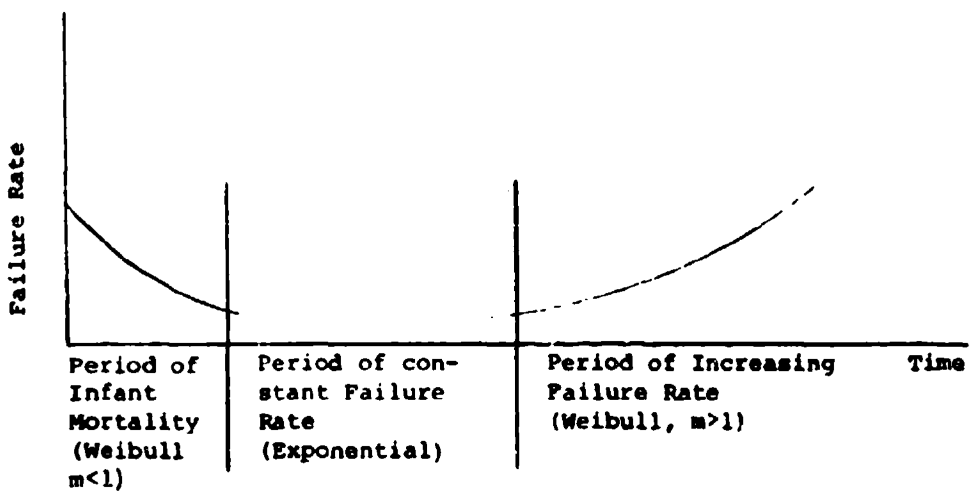

Figure 4. Failure Rate Curve

If in the actual verification test it can be shown that the distribution of failures for a given FLU is represented by

the Weibull distribution with m<1, then this observation, combined with a low MTBF would be symptomatic of a deficiency in equipment burn in.[1]

Since the procurement document does not address the maintenance and operations learning process, it has been assumed as stated in Chapter 1, that this learning process is not a significant factor. ARINC has recommended that the maintenance and operations personnel who participate in the six month test be, to the extent practicable, personnel who have acquired some experience with the equipment prior to the test. This recommendation, although well intentioned, will not obviate the problems of inexperience for two reasons. a. Not all maintenance and operations personnel will have the desired six months of experience. b. Six months of operational experience is not sufficient to reduce operator error to its final steady level. The term operator error is used here primarily to describe incorrect operator diagnosis which in turn impacts upon the "false pull" rate and ultimately upon the MTBF.

It is important to realize then that this learning process has been assumed away in this analysis and is not represented in the simulation model. If sufficient data were available to describe the process it could be incorporated into the model. The relative rate at which

[1]The term "burn in" is commonly used as the equivalent of the term "break in" when speaking of avionics equipment.

technical learning occurs has been estimated by M.A. Wilson (65:435) as shown in Figure 5 below.

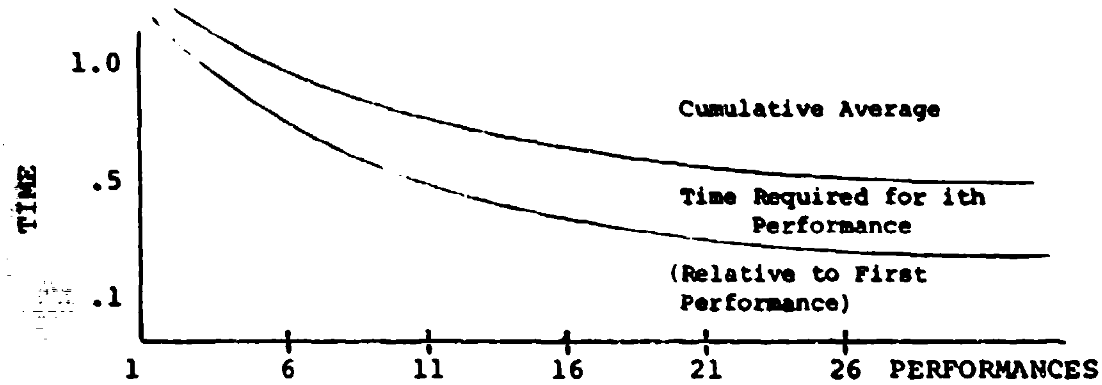

Figure 5. Maintenance Learning Curve

In order to incorporate such a model into the methodology given here, it would be necessary to know only the position on the abscissa for the average technician for a given task. As can be seen, the curve flattens out after 15 to 20 repetitions so that learning effects beyond this point may be considered negligible. The curve which Wilson derived above was for laboratory learning conditions, and it would be expected that field conditions would vary somewhat from these.

Regarding this aspect of the contract, ARINC has made the following observation.

> The contractor will be inclined to contest a declared deficiency centering on man hours required to perform maintenance due to inexperience or errors on the part of Air Force maintenance technicians. Ultimately,...., deficiency of man hour expenditure must be resolved either to a specific deficiency of the contractor's test procedures, test equipment, etc. or to an error or lack of experience on the part of Air Force maintenance [17:A-10].

The government will prepare a detailed test plan to assure that all data necessary to compute the appropriate support costs will be collected. The government will be responsible for all organizational, intermediate, and depot level maintenance and supply support for the verification test.

Provision is made in the contract for retesting in case correction of deficiencies action is taken. That is, following the correction of deficiencies, the government intends to verify, through such additional testing as it may deem necessary, that the TLSC-COD has been achieved for the control FLUs selected for COD coverage. The contractor obligation under this provision shall continue untill satisfactory compliance is demonstrated.

During the verification test, the contractor will provide representatives to verify the authenticity of the observed data. Representatives will also be provided for a retest should one become necessary. In the event a retest is required, the price for that portion of the test which is conducted in the contractor's facility will be separately negotiated.

Those items of the LSC model which were subject to verification were listed and defined in Chapter 1. They are listed again in Table III together with their appropriate measurement methods.

TABLE III

MEASUREMENT METHODS

FACTOR	MEASUREMENT METHOD
N, QPA, K	Direct observation
UC, CAB, CAD	The average negotiated unit prices in effect at the time of the test.
MTBF	The total reported flying time during the test period times the QPA divided by the number of failures for each FLU. For failure definition see Note 3 below. It is to be noted that the MTBF established in this manner is expressed in flying hours. As such, the value of 1/MTBF defined in this manner will be analogous to the expression UF/MTBF which was originally predicted. No UF factor will be established directly. (UF is ratio of operating hours to flying hours.
RIP, RTS, NRTS, COND	Observed fraction of total failures repaired in place, in baselevel shops, at the depot, or condemned at base respectively, as averaged over the test period.
(ALL MAINTENANCE MAN HOUR VALUES ASSOCIATED WITH FLU AND SYSTEM LEVEL MAINTENANCE PAMH, IMH, RMH, BMH, DMH)	Reported man hour expenditures against appropriate when discovered, how-malfunctioned and action taken codes (IAW AFM 300-4). The recorded values will be averaged over the test period.)
DOWN	Reported down time for peculiar support equipment (Average over test period).
APPLICABLE TO TLSC SYSTEM ONLY	
FLA, BA	The average negotiated costs for equipment necessary to support each squadron and base level shop respectively.
SMI	The scheduled maintenance interval prescribed by the appropriate technical order.

Note 1: Only the first 13 parameters are used in the control FLU LSC model.

Note 2: FLUs subject to verification will be those installed in the production aircraft delivered to the combat crew training squadron as well as any replacement spares delivered to support supply and maintenance requirements. These components will

undergo normal acceptance testing. No extraordinary qualification testing will be authorized. All organizational and intermediate maintenance shall be performed by Air Force using command personnel. Depot maintenance will be performed by the designated air logistics center to the maximum extent practicable. For control FLUs under COD the contractor is allowed the right for inspection for all government identified failures. For COD control FLUs, no maintenance at base level will be performed except removal and replacement until a contractor representative is present, provided he is present within 24 hours of notification. This provision does not apply to non control FLUs.

Note 3: The definition of a failure will be consistent with that used for reporting and consolidation under the AFM 66-1 maintenance data collection system. A failure shall be considered as any departure from the required performance in excess of the allowable tolerances defined in the appropriate configuration item specification.

It should be noted here that a removal which is found to be serviceable at bench check or depot shall still be deemed a failure if the erroneous failure identification is due to inadequately described test procedures or test equipment developed, procured, or prescribed by the contractor.

Failures caused by fire, explosion, or aircraft crash are exempt.

Summary of AFLC Logistics Support Cost Model and Discussion of Relevant Costs

The general approach in this section is to attack the AFLC logistic support cost model on a term by term basis. An explanation of variable names is included as Appendix F. As each term of the model is presented, an explanation of why it is or is not included in the TLSC-COD and TLSC-system computations is offered. For those costs whose inclusion is subject to controversy an expanded discussion of relevance is presented.

The computerized AFLC logistic support cost model consists of ten equations, each of which describes a portion of the resources required for an operating logistics

system. The model provides a method for estimation of the expected support costs which may be incurred by adopting a particular system. It is used to compare and discriminate among design alternatives where relative cost difference is the desired measure of merit. The significant result then is not the absolute value of the output but rather the difference between competing alternatives.

The following assumptions which are fundamental to the model must be considered in using the results.

1. The model assumes a uniform level of activity at each base.

2. The stock level for spares and pipeline quantities are computed to support the peak level of program activity. No provision is made for incremental build up.

3. The model explicitly computes only those logistic support costs associated with the weapon system, system, and first line units. Components below FLU level are considered only implicitly.

4. The model assumes one depot repair location and any given number of imtermediate level repair locations.

5. Quantities of support equipment are based on a manhour/machine hour equivalence. In other words, the model assumes that a given piece of support equipment is in use during the entire elapsed time period over which labor (in manhours) is required to perform a task.

6. Certain elements of resource consumption for which there is no basis for estimation, are not included. Examples

are, modification costs, and cost of maintenance actions generated by false pulls.

The first eight equations of the LSC model are structured to aggregate the cost of each system within the weapon system including subordinate FLUs and support equipment. Equations 9 and 10 compute propulsion system costs. The ten equations are described below.

1. SPARES

The first equation, C_1, is the cost of spares (initial and replacement).

(3-1) C_1 = cost of spare FLUs

$$= M \sum_{i=1}^{n} (STK_i)(UC_i) + \sum_{i=1}^{n} (PFFH)(QPA_i)(UF_i)(1-RIP_i)$$

$$\cdot \frac{(NRTS_i)(DRCT_i)(UC_i)}{MTBF_i}$$

$$+ \sum_{i=1}^{n} \frac{(TFFH)(QPA_i)(UF_i)(1-RIP_i)(COND_i)(UC_i)}{MTBF_i}$$

The first term in (3-1) is the investment in base stock. The second term is depot repair pipeline stock. This second term is essentially the peak number of failures per month, (PFFH/MTBF$_i$), times the fraction which are shipped to depot times the depot repair cycle time. This equation obviously will provide for an overabundance of pipeline spares in those months when flying is conducted at less than peak level.

The third term represents the total number of condemnations which will be replaced over a total force flying

program of 2,411,130 hours.

The entire C1 equation is applicable to both TLSC system and TLSC-COD. The fraction of failures condemned (COND) however is zero for all control FLUs which effectively eliminates it from the equation.

The computation of STK, for the first term involves the computation of a mean demand rate per base where

$$\lambda = \frac{(PFFH)(QPA)(UF_i)(1-RIP_i)}{M \cdot MTBF_i} \quad 2$$

So Lamda represents the average demand per base during a peak flying level month. Next, a weighted pipeline time must be computed. That is:

$$t_i = (RTS_i)(BRCT_i)+(NRTS_i)[(OSTCON)(1-OS)+(OSTOS)(OS)]$$

This is the average amount of time at base level to cycle an item from failure back to serviceability and installation. The second part of the term provides for overseas shipping of an appropriate fraction of the items.

So the product of λ_i and t_i is:

$$(\lambda_i \frac{\text{FLUs}}{\text{month}})(t_i \text{ months pipeline time}) = \begin{Bmatrix} \text{Expected No. of} \\ \text{Demands over average} \\ \text{Repair Pipeline Time} \end{Bmatrix}$$

The actual stock level STK is established by the following inequality.

Find the minimum value of STK_i such that

$$\sum_{x>STK_i} (x-STK_i)P(x|\lambda_i t_i) \leq EBO,$$

[2] In all equations, the subscript i represents the ith FLU.

where EBO is a government established acceptable expected back order level and the distribution of probabilities of demand is a Poisson distribution with mean $(\lambda_i t_i)$. When the equation is solved for the minimum value of STK_i, then the cost for the ith FLU at all bases is $(M) \cdot (STK_i) \cdot (UC_i)$.

The Poisson distribution, Figure 6 below, is quite commonly used to represent inventory demand rates. Its use required only the assumptions that: 1. The likelihood of the occurrence of a demand in a given period does not change over time. 2. That the occurrence of an event has no effect on whether or not a subsequent event occurs (31:38).

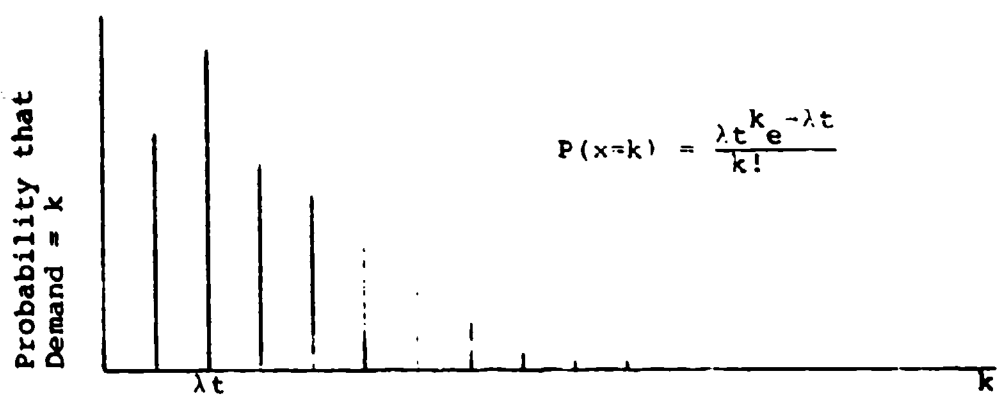

Figure 6. The Poisson Distribution with mean λt

The expected back order (EBO) term is a constant which is determined by Air Force policy. It is of interest, however, to examine the behavior of logistic support cost as a function of changes in EBO. To this end, a sensitivity analysis has been performed on EBO as is shown in Appendix C. This sensitivity analysis shows that for values of EBO less than approximately .05, the logistic support cost increases

rapidly. For such low values of EBO, a large number of spares would be required. In other words, it would become very expensive to reduce the risk of shortage much below .05. Decreasing EBO from .1 to .05 would cost about $2.6 million, while decreasing it from .05 to .03 would cost about $3.0 million.

Increasing the value of EBO above .1 and accepting greater risks of shortage would have very little effect on logistic support cost. For example, increasing EBO to .2 would save about $2.4 million. A further increase from .2 to .3 would save only about $2.0 million.

In summary, small reductions in EBO (down to .05) could be made at a relatively modest cost, while further reductions would soon become prohibitively expensive. Fairly large increases in EBO are required to obtain substantial decreases in logistic support cost. The value of .1 used in this analysis seems well chosen, though it would be reasonable to consider reductions in the range of .05 to .1.

2. ON EQUIPMENT MAINTENANCE

The second equation in the LSC model, C_2, represents the cost of on equipment maintenance, i.e.,

(3-2) C_2 = Cost of on equipment maintenance[3]

$$= \sum_{i=1}^{n} \frac{(TFFH)(QPA_i)(UF_i)}{MTBF_i} [PAMH_i + (RIP_i)(IMH_i) + (1-RIP_i)(RMH_i)](BLR) + \frac{TFFH}{SMI}(SMH)(BLR)$$

$$+ \boxed{\frac{(TFFH)(EPA)(ERMH)(BLR)}{CMRI}}$$

The first term im (3-2) is the expected cost of unscheduled in place maintenance over the lifetime of the equipment. It includes allowance for manhours expended in preparation and access activities, PAMH, actual in place maintenance activities (IMH), and removal or replacement activities (RMH).

The second term provides for scheduled maintenance manhour costs over the equipment lifetime.

The third term applies only to power plant systems. It accounts for the maintenance manhours required for removal and replacement of engines over the total force flying hours.

In the present application, the first term of 3-2 is applicable to both TLSC-system and TLSC-COD. The second term is applicable to TLSC-system only. The third term, engine maintenance, is not used here as it is not applicable.

[3] All engine related terms will be enclosed in dotted lines as above. These terms are not relevant to this analysis.

Serious questions have been raised regarding the relevance of the cost element C_2 to a decision between competing alternatives or to an application involving measurement of logistics supportability. Consider what 3-2 is measuring. The equation multiplies the aggregate manhour expenditure by the base labor rate to determine an overall cost. This base labor rate is a summation of the estimated hourly wage of an average Air Force skill level plus the pro rata share of base level costs required to support that technician. It is important to emphasize at this point that a life cycle cost measurement system ought to exclude any fixed costs which would be incurred whether a particular system is procured and operated or not (24). If a particular cost will be incurred regardless of what choice is made between alternatives or regardless of whether or not a system is bought and operated, then that cost is irrelevant to any life cycle cost decision making, and it should be excluded from the model. It could be argued that for policy or political reasons many base level costs would continue even if a new system such as the F-16 were not procured and operated at those bases. It could also be argued that a given reduction in manhour requirements for a weapon system does not always result in the same proportional reduction in unit manning levels. These manning levels may be determined more directly by policy and contingency plans than by current requirements. These are very difficult questions and good arguments can be made for both sides. One important

point is offered here in support of the relevance of these costs.

If the assertion that manning levels are determined by policy rather than actual hardware requirements is accepted at face value, then the next logical question is: How are the policies for manning levels established? It is reasonable to assume that these manning levels would be established in response to some contingency plan which required a work force in excess of that required for routine operations and perhaps even in excess of that required to support the peak force flying hours. Further, it seems safe to assume that there is some relationship between policy established manning levels and actual hardware man hour requirements. In this light then, even though a unit might have an authorized to required ratio of say 1.25/1 it would be expected that, in the long run, as it becomes apparent that improvements in reliability and maintainability have truly occurred, the policy makers would reduce the ratio somewhat. This theory is particularly believable in that there is general recognition of the fact that manpower costs account for a very large portion of the DOD budget. In the short run, the ratio could be reduced by retraining of excess technicians into weapons systems which have shortages. In the long run, the ratio would be reduced by cutbacks in recruiting. In summary, it is probably true that manning levels are not directly established by weapons systems manhour requirements, but at the same time it is reasonable to expect that

in the long run the manning levels will be responsive to requirements. The conclusion offered here, then, is that the manhour costs, together with the pro rata share of base services are a relevant cost in life cycle cost calculations.

3. OFF EQUIPMENT MAINTENANCE

The third equation in the model represents off equipment maintenance costs, i.e.,

(3-3) C_3 = cost of off equipment maintenance

$$
\begin{aligned}
&= \sum_{i=1}^{n} \frac{(TFFH)(QPA_i)(UF_i)(1-RIP_i)}{MTBF_i} \{(BCMH_i)(BLR) \\
&\quad + RTS_i[(BMH_i)(BLR + BMR) + (BMC_i)(UC_i)] \\
&\quad + [2(NRTS_i) + COND][(PSC)(1-OS) \\
&\quad + (PSO)(OS)](1.35\ W_i)\} \\
&\quad + \boxed{\frac{(TFFH)(EPA)(1-ERTS)(EOH)(EUC)}{CMRI}}
\end{aligned}
$$

The first term in 3-3 describes the cost of maintenance for those FLU failures which are not repaired in place. This term is applicable to both TLSC-system and [illegible]. For those failures which are reparable on station on the equation finds the sum of the costs for bench checking (BCMH) and for direct repair manhours (BMH). Also included are the implied cost of stock and repair of lower level sub assemblies (BMC). Costs incurred at depot level are summed in the same equation in an exactly parallel fashion. Also included are the costs of shipping and depot

level condemnations. The last term applies only to powerplant systems and is not included in this analysis.

The argument for relevance of manhour costs in this equation is exactly the same as that for on equipment maintenance, and it will not be restated here.

4. INVENTORY MANAGEMENT

The fourth equation is inventory management cost, i.e.,

(3-4) C_4 = Inventory management cost

$$= [IMC + (PIUP)(RMC)] \sum_{i=1}^{n} (PA_i + PP_i + 1)$$

$$+ (M)(SA)(PIUP) \sum_{i=1}^{n} (PA_i + PP_i + SP_i + 1)$$

The first term in 3-4 is the cost for entering an item into inventory and maintaining it over the system life. The second term accounts for the base level inventory costs.

This equation is excluded from both TLSC-system and TLSC-COD calculations. It is considered a negligible cost.

The same general questions of relevance arise with this equation as with C_2 and C_3. No detailed discussion is offered here since the equation is not used in this application It is sufficient to say, that in any application, the relevance of the cost ought to be evaluated in terms of whether it is a cost which can be avoided if the system is not procured and operated. This evaluation should consider the long run as well as the short run.

5. SUPPORT EQUIPMENT

Equation five is the cost of support equipment, i.e.,

(3-5) C_5 = Cost of support equipment.

$$= \sum_{i=1}^{n} \frac{(PFFH)(QPA_i)(1-RIP_i)}{MTBF_i} \sum_{j=1}^{k} \frac{(RTS_i)(BMH_i+BCMH_i)}{(BUR_j)(BAA)(1-Down_j)}$$

$$[1+(PIUP)(COB_i)]CAB_j + \frac{(NRTS_i)(DMH_i)}{(DUR_i)(DAA)(1-Dcwn_j)}$$

$$[1+(PIUP)(COD_j)]CAD_j + [1+.1(PIUP)][DCA+DPA$$

$$+ M(BCA+BPA+FLA)] + CS + IH$$

The first term in 3-5 calculates the acquisition and maintenance costs for that support equipment which is required for support of FLUs. Workload, usage rate of support equipment, and down time of support equipment are all considered in this calculation.

The second term accounts for support equipment which is non-workload related.

The cost element C_5, is zero for all control FLUs since there is no peculiar item of support equipment required for any control FLU. Equation C_5 is present in TLSC-system calculations.

6. PERSONNEL TRAINING

Equation six is the cost of personnel training i.e.,

(3-6) C_6 = cost of personnel training.

$$= \frac{[1+(PIUP)(TRB)]TCB}{(PIUP)(PMB)} \left\{ \sum_{i=1}^{n} \frac{(TFFH)QPA_i)(UF_i)}{MTBF_i} \right.$$

$$\left\{ PAMH_i + (RIP_i)(IMH_i) + (1-RIP_i)[RMH_i + BCMH_i + (RTS_i)(BMH_i)\right\}$$

$$\left. + \frac{TFFH}{SMI}(SMH) + \boxed{\frac{TFFH)(EPA)(ERMH)}{CMRI}} \right\}$$

$$+ \frac{1+(PIUP-1)(TRD)}{(PIUP)(PMD)} \quad TCD \sum_{i=1}^{n} \frac{(TFFH)(QPA_i)(UF_i)}{MTBF_i}$$

$$(1-RIP_i)(NRTS_i)(DMH_i) + TE$$

The first and second terms in 3-6 are respectively training costs at base and depot level. The equations first determine life cycle manhour requirements. For example, depot life cycle manhour requirement =

$$\frac{(TFFH)(QPA_i)(UF_i)(1-RIP_i)(NRTS_i)(DMH_i)}{(MTBF_i)}$$

Dividing this term by (PIUP) times (PMD) gives personnel required at depot. Multiplying this by (1+(PIUP)(TRD)) accounts for personnel turnover. All of this is then multiplied by TCD, the cost of training one man.

Cost Element C_6 is excluded from both TLSC-COD and TLSC-system, because it is considered a negligible input. In those applications where it is not negligible, the same questions of relevance as those discussed in relation to equations C_2 and C_3 must be answered. If Air Force policy requires the number of personnel assigned to the system to be in excess of actual requirements then the

relevance of the personnel costs is not totally clear.

7. MANAGEMENT AND TECHNICAL DATA

Equation seven accounts for the costs of management and technical data, i.e.,

(307) C_7 = cost of management and technical data

$$= \sum_{i=1}^{n} \frac{(TFFH)(QPA_i)(UF_i)}{MTBF_i} [MRO+(1-RIP_i)(MRF+SR+TR)]BLR$$

$$+ \frac{TFFH}{SMI} [MRO+.1(SR+TR)]BLR+TD(JJ_{th})$$

The first term here is the manhour cost of completing forms and administrative tasks during routine maintenance. The second term similarly accounts for administrative costs during scheduled maintenance. The last term represents the purchase cost of technical data. Cost element C_7 is considered negligible and is omitted from TLSC-system and TLSC-COD computations.

8. FACILITIES

Equation eight is the cost of facilities, i.e.,

(3-8) C_8 = cost of facilities

$$= FD + (M)\ (FB)$$

These terms describe the costs of new or special facilities for base or depot support. These costs are relevant to the decision only if they would not be incurred if the system were not procured and operated. As has been pointed out by Hitch and McKean (32:138), the allocation of this entire cost to the weapons system in question

implicticly assumes that these facilities depreciate at the same rate as the weapons system. This may not always be true, and yet it may be necessary to make the assumption simply because there is inadequate supporting data for making a more accurate assumption. It is well to note that these same comments could be applied to personnel training as well as facilities since this training presumably has some finite life and some salvage value.

In this application the costs of new facilities are held to be negligible, and equation eight is omitted from both TLSC-COD and TLSC-system.

9. ENGINE RELATED COSTS

Equation nine is fuel cost, i.e.,

(3-9) C_9 = cost of fuel consumption

= (TFFH)(EPA)(FR)(FC)

This equation calculates the life cycle fuel costs. It is not applicable to either TLSC-COD or TLSC-system.

10. SPARE ENGINES

Equation ten is the cost of spare engines, i.e.,

(3-10) C_{10} = cost of spare engines

= ((LS)(X) + Y EUC

where X is base stock and Y is depot pipeline spares. The value of X is determined by the mean demand rate $\frac{(PFFH)(EPA)}{(LS)\ (CMRI)}$, by the weighted base pipeline time (ERTS)(BP) + (1-ERTS)(ARBUT) and by CONF, the policy established confidence level of availability. X is the minimum value which satisfies

$$\sum_{n=0}^{x} \frac{(e^{-ARGB})(ARGB)^n}{n!} \geq CONF$$

Here ARGB is the mean of a Poisson distribution of demand. ARGB is the product of demand rate and base pipeline time. So ARGB represents the mean number of demands per pipeline cycle time.

Equation ten is, of course, not applicable to TLSC-COD or TLSC-system.

A summary of applicable costs is presented in Table IV.

TABLE IV

APPLICABLE COSTS

TLSC-COD	TLSC-SYSTEM
C_1 - spares	C_1 - spares
C_2 - on equipment maintenance	C_2 - on equipment maintenance
C_3 - off equipment maintenance	C_3 - off equipment maintenance
	C_5 - support equipment

This chapter has discussed the important details of the F-16 COD incentive contracting provisions as well as the generalized AFLC Logistics Support Cost Model. Both of these subjects are prerequisites to a thorough understanding of the analysis which is to follow. The presentation of the AFLC LSC model should facilitate a generalization of the total methodology to be developed here.

In the next chapter, the analytical methodology is developed.

Chapter 4

THE ANALYSIS

Introduction

In this chapter, a complete description of the mathematical analysis is presented. Those concepts which would not be universally understood are explained in an expository fashion. The mathematical details which are not essential to an understanding of the material are omitted from this chapter and included instead in Appendix D where the interested reader can verify the correctness of the complete methodology.

In order to avoid any misunderstanding or inappropriate extrapolation of the results of this study, the fundamental assumptions upon which the study rests will be carefully explained at the onset. Later, the effects of relaxing some of these assumptions are examined. It is important to recall, that in addition to the assumptions listed here, the study is also dependent upon those assumptions already described upon which the logistic support cost model is based.

Use of Probability Distributions

Some mention has already been made of the probability distributions which will be used here in an attempt to represent reality. Before going further it is appropriate

to more carefully describe the concept of probability distributions. If a set of events has some theoretical distribution, as depicted in Figure 7 below, it can be said, for example, that about 34 per cent of the observations sampled from that population will fall in the interval AB. Or, equivalently, it can be said that the likelihood of any one observation falling into that interval is .34. Similar statements can be made regarding the probability that a single observation from the distribution will fall into any given interval.

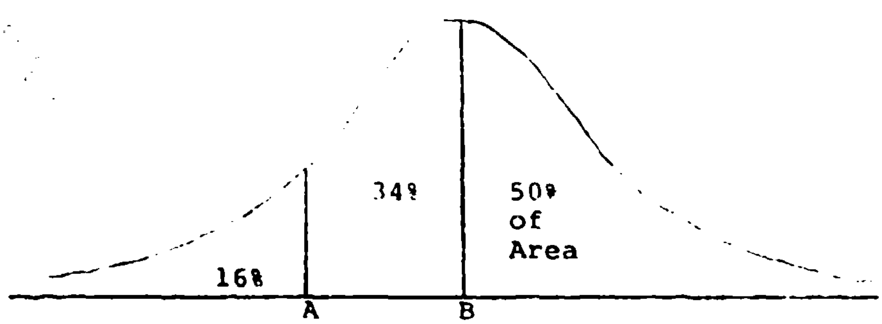

Figure 7. Probability Distribution

The distributions which will frequently be used in this analysis will be cumulative distributions. A cumulative distribution function (CDF) will depict, for any given value, the fraction of observations which will be either less than or equal to that value. In Figure 8 below, for example, the point A is that value below which about 16 per cent of the observations will fall. The point B is that point below

which 50 per cent of the observations will fall.

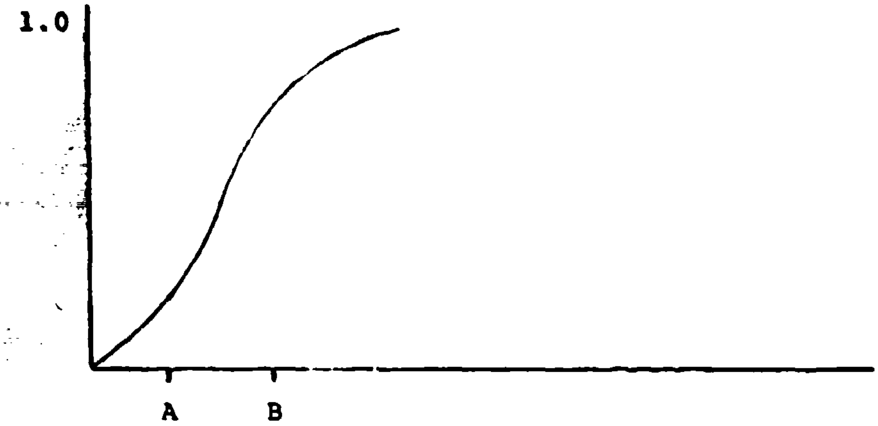

Figure 8. Cumulative Probability Distribution

This analysis is based on three probability distributions. Both the theory, and some empirical data support the assertion that these distributions are a fair representation of the real world distributions which they are supposed to describe. Of course, if a real world distribution can be adequately described by existing data, then that real world (empirical) pdf ought to be used. As stated earlier, however, insufficient data exists to construct empirical distributions for any of the activities addressed in the analysis. Hence, the actual approach used here has been to first determine what distribution the theory would suggest. This theory is then checked against reality by determining what distributions provide the best representation of applicable past experience. Finally, if the real world data suggests some uncertainty as to the correct representation, then a sensitivity analysis is conducted

to determine the impact of an incorrect choice of distributions.

Time to Failure Probability Distribution

Consider first the probability distribution of the lifetimes of each FLU. It is commonly assumed that the failure rate, λ, for electronic equipment, is not a function of the age of the equipment. That is, if the equipment has been operated for 50 or 500 hours, its probability of failure in the next instant is the same as that for a piece of equipment which has operated only for two hours. If this constant failure rate can be accepted as a fair representation of the failure characteristics of the equipment, then it can be shown mathematically that this equipment has an exponential distribution of life times.[1] Figure 9 depicts the exponential distribution.

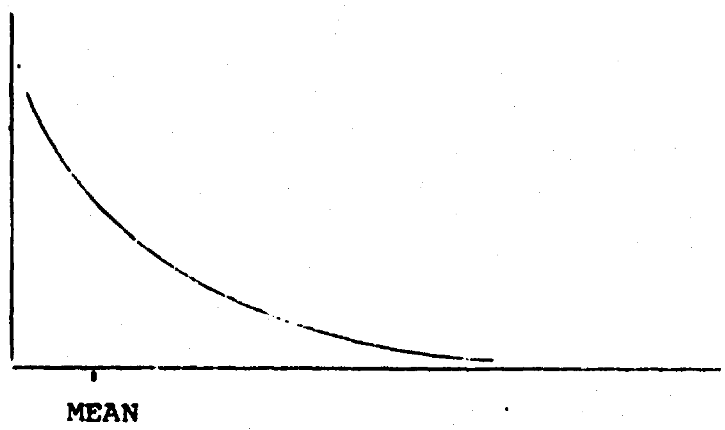

Figure 9. The Exponential Distribution

[1]See Appendix D for proof.

In order to check this assumption of exponential lifetimes against past history, the records of Category II reliability and maintainability tests have been consulted. These documents provide carefully measured empirical data which has been "fitted" by statistical methods. A total of eleven avionics equipments from the A7D Category II tests were checked (49). The results are shown in Table V below.

TABLE V

DISTRIBUTION OF LIFETIMES
Airborne Electronic Equipment
A7D

Total avionics equipments 11

Distribution	Number
Exponential time to failure	6
Weibull time to failure	2
Lognormal time to failure	3

Of these 11 equipments, four are the actual systems upon which the F-16 control FLU reliability predictions are based. Of this group of four, two are exponential, one Weibull and one lognormal. Of course, this sample is not adequate alone to provide convincing evidence of the correctness of the exponential time to failure. The assumption is based on three points all taken together:

1. The theory strongly supports the exponential assumption.

2. The A7D data which is taken from similar equipment operating in a fighter environment tends to support th

theory.

3. Sensitivity analysis on the uncertainty of LSC as a function of MTBF shows that an assumption of Weibull failures yields results which are not significantly different from the exponential assumption at the 95 per cent confidence level.[2]

Time to Repair Probability Distribution

The next probability distribution to be considered is the distribution of time to repair. As was stated earlier, the intention here was to find one type of distribution to represent maintenance activities on the flight line, at base level, and at depot. The literature of maintainability is almost unanimous in the belief that this distribution is lognormal. From a purely theoretical standpoint, there is strong support for the lognormality. Behavioral studies have shown that when human beings participate in a complex task which involves classification and categorization, the probability distribution of times to complete the task is best represented by the lognormal (26:54). Since avoinics trouble shooting and repair are tasks which involve the use of test equipment to isolate and categorize faults, it is not surprising that these tasks would have the lognormal distribution. The asymmetry of this distribution is

[2]For this test a Weibull distribution with shape parameter = 1.1 was used, simulating an equipment whose failure rate would increase slightly over time.

intuitively appropriate as a representation of time to repair. It would be expected that most repair times would be clustered around some relatively low value, with a few observations (for example, when no fault is located) quite far out to the right. There are other distributions which possess this asymmetry, but none which derive from the categorization process described above (26:55). The distribution is shown in Figure 10 below.

Figure 10. The Lognormal Distribution

The amount of skewness is controlled by the variance of the distribution. In order to compare the theory above with real world observations, the Category II maintainability tests were reviewed for consistency.[3] First, considering the A7D data mentioned above; of the 11 equipments, five were lognormal, four were Weibull, and two had no fit. These data alone would seem to provide a somewhat weak confirmation of the theory. Of those four equipments

[3]These data were taken from Air Force Flight Test Center Category II Reliability and Maintainability Test reports (49; 50; 51).

which were baseline for the F-16, two were lognormal and two Weibull. Since other maintainability data were available from recent Category II tests on F-111 and C-5A avionics, those data were also reviewed. The results for 34 avionics equipments are summarized in Table VI below.

TABLE VI

MAINTAINABILITY DATA

Best Fit	Number
Lognormal	18
Weibull	7
Exponential	2
No Fit	6
Total	34

Of 34 modern avionics equipments, then, 53 per cent had a lognormal time to repair, with the remainder scattered between the Weibull, the exponential, and no fit. This is strong evidence for acceptance of the lognormal distribution.

Specifying the correct lognormal distribution is somewhat more complicated than specifying a correct exponential distribution because the lognormal is a two parameter distribution whereas the exponential is a one parameter distribution. In the case of the exponential, the standard deviation was specified at the time as the mean, because the exponential standard deviation is equal to the mean. No particular relationship exists between the mean and variance

of a lognormal distribution. To solve this problem a review of the standard deviation to mean ratio of the 18 observed lognormal distributions above was undertaken. It was determined that the average standard deviation to mean ratio was 1.2. For both fighter types of aircraft, (A7D and F-111) the ratio was very close to 1.0 (.929 and .975 respectively). After considering all of the above information, a standard deviation to mean ratio of 1.0 was assigned for all lognormal maintenance tasks.

Probability Distribution for Fraction Reparable this Station

Determination of the correct distribution to represent the reparable this station (RTS)/not reparable this station (NRTS) decision is a somewhat different matter. This decision is what is often called a Bernoulli trial in which one of two outcomes must occur. A toss of a biased (or unbiased) coin is the usual example of a Bernoulli trial. This process is represented by the binomial distribution. The following formal assumptions are necessary to satisfy the binomial distribution.

1. The experiment consists of n identical trials.

2. Each trial results in one of two outcomes, usually called success or failure.

3. The probability of success, p, is constant from trial to trial. The probability of failure is (1-p)=q.

4. The trials are indopendant.

If RTS is labeled as a success and NRTS as a failure (or

vice versa) then it is clear that this process will satisfy the assumptions of the binomial distribution. It is, of course, possible that the value of p might be changed over time by modifications to the equipment, but it seems safe to assume that its true value will be nearly constant over a 3500 hour test.

Selection of the Analytical Approach

In this thesis, the simulation approach has been used to develop a relationship between the stochastic input parameters and the probability distribution of MLSC's. In the following discussion, the drawbacks associated with the other approaches are described.

It was stated earlier that there are several methods which could be used to attack this problem. In fact, one of these methods, error theory approximation, was used in initial attempts in this thesis. It was found in these attempts that this method breaks down when attempting to deal with the stock level calculations. The following derivation will serve to illustrate some of the difficulties.

Using error theory, the purpose is to find the variance of MLSC as a function of the variance of MTBF, RTS, and MMH.

First, it is true that:

$$V(MLSC)=V(C_1)+V(C_2)+V(C_3)+V(C_5)$$
$$=V(C_{11})+V(C_{21})+V(C_{31})+V(C_{51})+\ldots$$
$$\ldots+V(C_{1,13})+V(C_{2,13})+V(C_{3,13})+V(C_{5,13})$$

Considering each FLU there are 52 such terms. Almost every one of these terms contains MTBF, MMH, and RTS. For each of these terms in which there is a common random variable which cannot be factored out, a covariance term develops. There could be a very large number of these covariance terms.

To simplify the problem, assume RTS and MMH are not random variables. Now MTBF can be factored from each of the equations. Considering only equation C_1 for simplicity:

$$C_1 = (STK_i)\ (M)\ (UC_i)$$

$$+ \frac{1}{MTBF} \sum_{i=1}^{N} (PFFH)(UF_i)(QPA_i)(1-RIP_i)(NRTS_i)$$

$$\cdot\ (DRCT)\cdot(UC_i)$$

(After deleting the second term since COND=0 as explained earlier.)

And dealing with one FLU for simplicity:

$$V(C_1) = V[STK\cdot M\cdot UC]+V\left[\frac{1}{MTBF}\right][PFFH\cdot UF\cdot(1-RIP)\cdot NRTS$$

$$\cdot\ DRCT\cdot UC]^2$$

$$+\ COV\left[(STK\cdot M\cdot UC),\ \frac{PFFH\cdot UC\cdot(1-RIP)\cdot NRTS\cdot DRCT\cdot UC}{MTBF}\right]$$

This co-variance term arises since STK is a function of MTBF.

Now looking only at V(STK·M·UC) and recalling the definition of STK: STK is the minimum value which satisfies the inequality

$$\sum_{x>STK} (X-STK)\ P(X|\lambda t) \le EBO$$

$$\text{where:}\quad \lambda t = \frac{PFFH\cdot UF\cdot QPA\cdot(1-RIP)}{MTBF\cdot M}\cdot[RTS\cdot BRCT$$

$$+NRTS[OSTCON(1-OS)\ +OSTOS\cdot OS]$$

The above equation must be solved explicitly for STK before the variance of STK can be written as a function of MTBF. Unfortunately it is not possible to solve the equation for STK while that term is present in both the index of the summation and the summation itself. The circumvention of this problem would by no means establish a clear path to an analytical expression, but it is not necessary to consider explicitly the further difficulties expected when a solution to the above problem is not apparent.

In further support of the simulation approach to the problem, the following observation by F.S. Timson is offered.

> The problem that there may be differences between the results obtained with the two methods (Monte Carlo vs. Error Theory) is that the probability distributions involved may not be normal. Monte Carlo can be used with probability distributions having any form while propagation of error requires that all distributions be normal; hence, in situations involving non normal distributions, Monte Carlo calculations with large samples should yield results that are closer to reality [61:123].

A final observation in favor of the simulation approach is that it can accommodate empirical distributions. This last point is important in that the best representation of reality would be derived from a simulation which used actual empirical distributions rather than theoretical approximations.

In summary, the advantages of simulation are: 1. Ability to circumvent mathematically intractable situations. 2. Greater accuracy when the underlying distributions are

non-normal. 3. Greater generality of application since virtually any sort of input distribution can be used.

Monte Carlo Simulation

As stated in the last section a simulation technique was used to develop an output distribution of MLSC's as a function of appropriate random inputs. It is appropriate now to clarify the inputs, the output, and the method used to develop the relationship. Figure 11 below is a reproduction of Figure 3 from Chapter 1 with some details added.

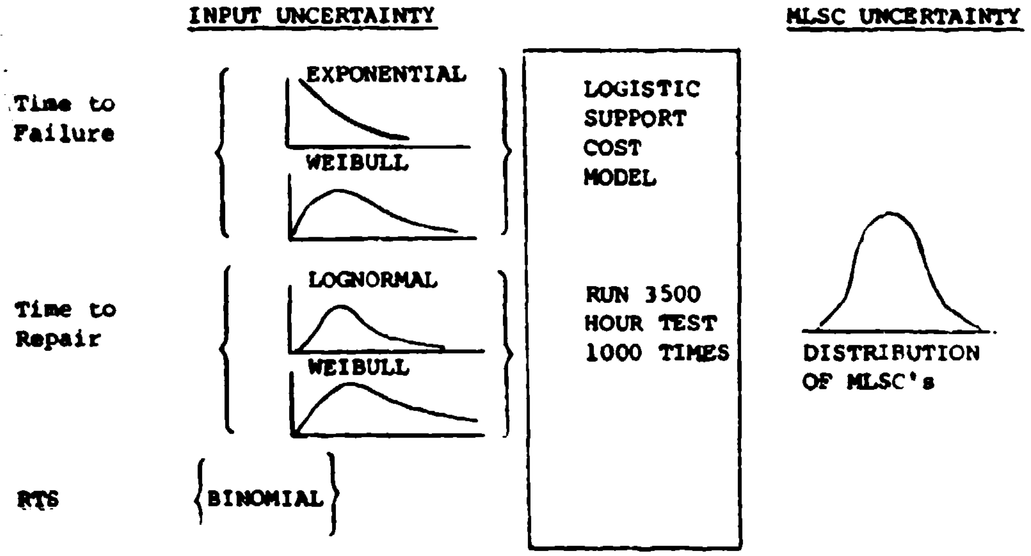

Figure 11. Uncertainty Sources

The first question which arises is how to simulate the input uncertainty which is known to be represented by the distributions above. The first step is to convert the continuous PDF's of the inputs into cumulative distributions. Taking the lognormal distribution as an example, the

cumulative distribution would have an appearance something like that shown in Figure 12.

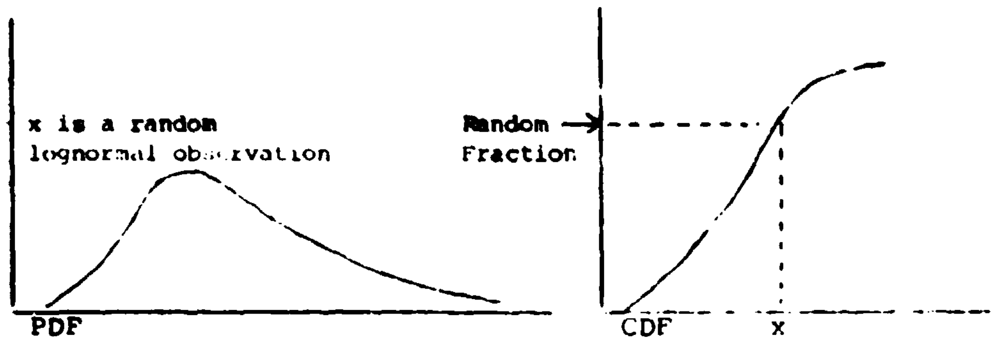

Figure 12. Random Sample From Lognormal Distribution

Next it is necessary to generate a random number input. Many computer routines are available to generate random fractions between 0 and 1.0.[4] Given a random fraction y, $0<y<1$, the point $x=F^{-1}(y)$ (where $F^{-1}(y)$ is the inverse function of the lognormal CDF, $F(x)$) represents a random orservation from the lognormal distribution, $F(x)$. If a large number of these samples are taken, say 1,000, and plotted in a histogram, that histogram would resemble very closely the lognormal distribution above. This then is the general method used to generate samples from a given continuous distribution.

The generation of random variables from the binomial distribution is somewhat simpler. Here it is only necessary to generate a random fraction and check to determine whether

[4]See Appendix D for discussion of random number generation.

it is above or below a given value. For example, assume an RTS value of .6. This process could be simulated by generating random fractions, and for all values $\leq$ .6, designate those observations reparable this station. All observations greater than .6 will be designated not reparable this station.

The mathematical device used to convert a random fraction to a random observation on a particular CFD is called a process generator. For the binomial distribution, the process generator is simply the random number generator itself. For the continuous distribution used here, the process generators are more complex. The derivations for these process generators are shown in Appendix D. It is sufficient here to simply list the generators to be used. They are listed in Table VII on the next page.

TABLE VII

PROCESS GENERATORS

DISTRIBUTION	GENERATOR
Exponential	Time to Failure=MTBF(Ln(RAND))
Lognormal	$V=((-2Ln(RAND1))^{.5})\cos(2\pi RAND2)$ $V1=V\cdot SD+\mu$ $Manhours=2.718^{V1}$
Weibull	Time To Failure$=\frac{MTBF}{\Gamma(1+1/\theta_2)}(Ln(RAND))^{1/\theta_2}$ Manhours$=\frac{\left\{\begin{array}{l}\text{Contract}\\\text{Mean}\\\text{Manhours}\end{array}\right\}}{\Gamma(1+1/\theta_2)}\cdot(Ln(RAND))^{1/\theta_2}$
Binomial	The Random Number Generator RAND is used to produce binomially distributed variates to stimulate RTS

Note 1: RAND is a random number generator producing random numbers uniform on the interval (0,1)

Note 2: RAND1 and RAND2 are independent random numbers, uniform on the interval (0,1)

Note 3: V and V1 are dummy parameters for the lognormal generator. μ and SD are the mean and standard deviation of the lognormal distribution.

Note 4: θ_2 is the shape parameter of the Weibull distribution.

Given a method for obtaining stochastic time to failure, manhours, and RTS the next step is to simulate the sampling of these values so that they would appear just as though they had been observations from a 3,500 flying hour test program.

From this point, to follow the logic of the computer model which was used it will be necessary to refer to Figure 13, the abbreviated computer flowchart. The complete listing of the program is included in Appendix D for the interested reader. In blocks 1 and 2 the computer reads all inputs including FLU data such as RMH, RIP, RTS, and user inputs such as choice of probability functions and stochastic variables. Assume here that the program chosen is for all three variables stochastically determined with exponential time to failure and lognormal maintenance activity. Given these inputs, the program flow is to Block 9 where exponential times to failure are generated. These lifetimes are recursively generated and added together until their sum is greater than 3,500. Then to find measured MTBF (TBFM), 3,500 is divided by the number of failures minus one (I-1). The rationale for this procedure is that in an actual 3,500 hour test there would probably be no failure at precisely the 3,500 hour point; rather, there would i-1 failures before the 3,500 hour point; and another (the ith) sometime after the 3,500 hour point. The actual procedure in the real test would be to divide 3,500 by the number of failures which had occurred up to that point or I-1. Of course in the simulation, failures must be generated up through the ith which is the first one to occur after 3,500 hours elapsed. There is no other way of determining I-1. If a Weibull failure rate were chosen, the determination of TBFM would proceed in an exactly parallel fashion.

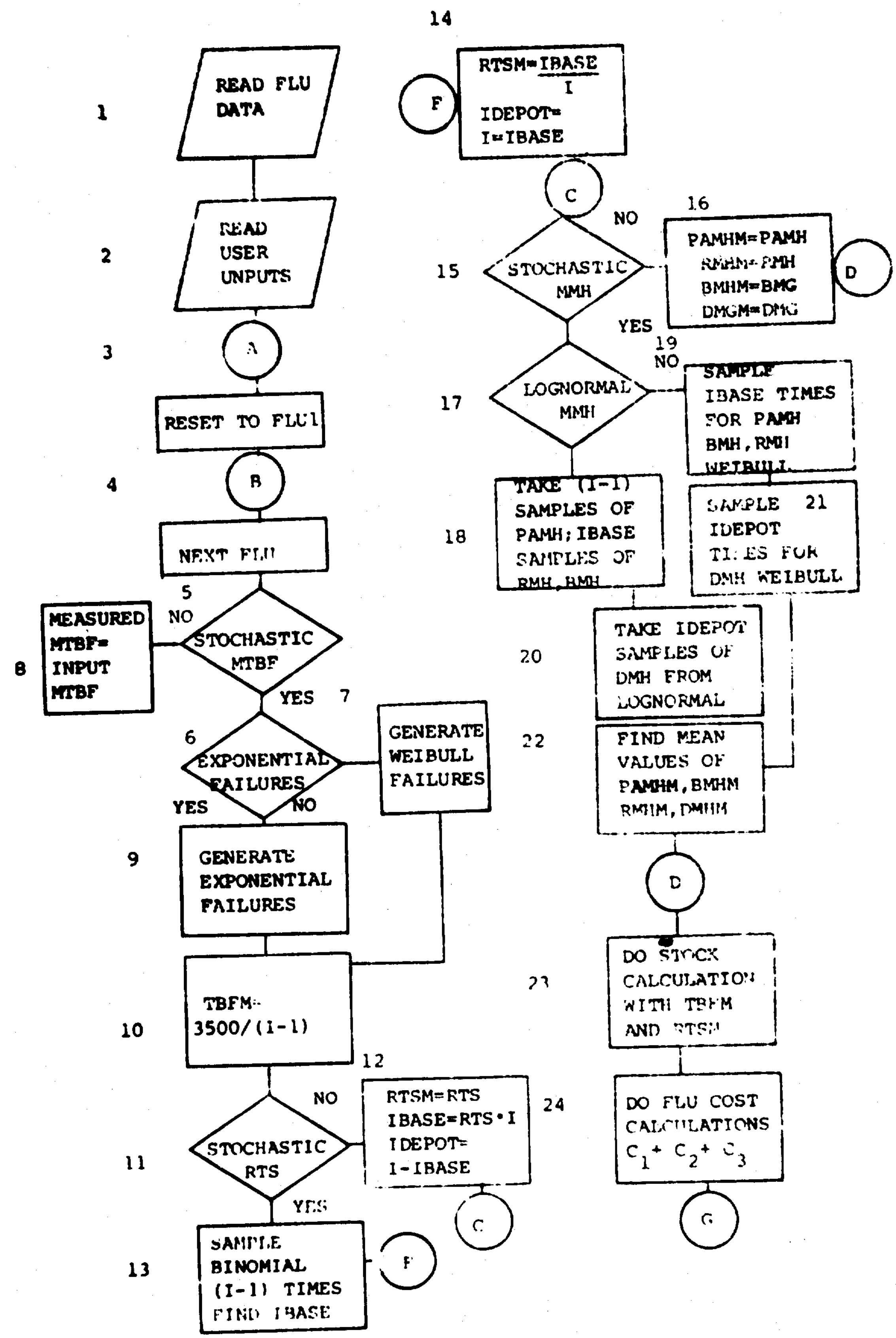

Figure 13. Abbreviated Computer Flow Chart

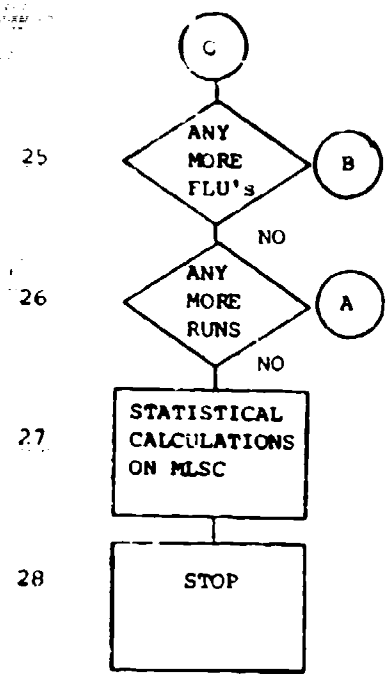

Figure 13. (continued)

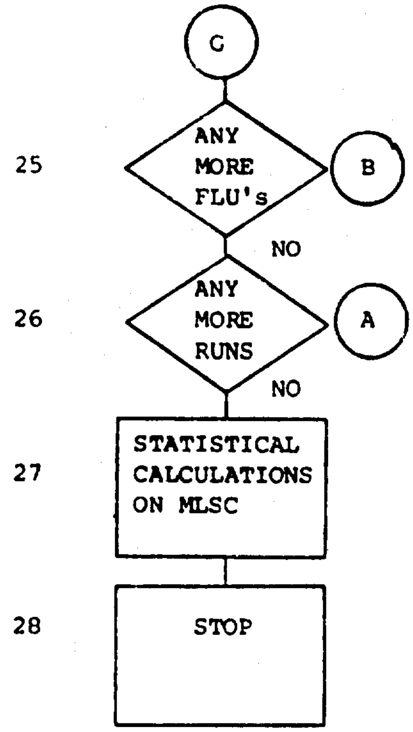

Figure 13. (continued)

Or if non-stochastic MTBF's are desired, then TBFM is set equal to TBF. (TBF is the computer variable name for contract specified MTBF.)

In the next step, Block 13, the measured RTS called RTSM is determined. Every failure which occurred during the test is subjected to the binomial sampling procedure, to determine whether it will be repaired at this station or sent to depot. After all (I-1) failures have been disposed of, then RTSM is just the number repaired on station divided by (I-1). Symbolically, RTSM=IBASE/(I-1). Since base level condemnations are zero here NRTSM is just 1-RTSM; and the number repaired at depot is just (I-1)-IBASE.

Once the numbers repaired at base and depot level are determined, then appropriate samples may be taken from each of the four manhour distributions, Block 18. For PAMH, there are (I-1) samples, since every failure generates preparation and access activity. For RMH and BMH there are IBASE samples taken. For DMH there are IDEPOT samples taken. Finally then in Block 22 the average values for each of these variables over the 3,500 hour test are found.

In Block 23, the stock level calculation is performed based on the measured values of TBFM and RTSM. When the stock level (STK) is known, then the program can compute the values of each of the cost equations, C_1, C_2, and C_3, based on the measured inputs for MTBF, MMH, RTS and STK. The sum of $C_1+C_2+C_3$ then is the measured logistic support cost (MLSC) for one FLU.

The same identical sequence then must be run for each of the 13 FLUs, each time adding an increment to the value of MLSC. When all FLUs are included, then the result is one sample point of MLSC.

Determining the Form of the MLSC Distribution

With this group of 1,000 observations of MLSC an empirical probability distribution of MLSC's may be constructed. To assist in this task, the computer model above has a statistical routine. This routine sorts all of the observed MLSC values into ascending order, calculates a frequency distribution based on 20 equal intervals, and prints a histogram of observations. Figure 14 below shows a sample of a typical histogram printout. Also included in the output are the mean and standard deviation of the sample of 1,000 MLSC's. With the above information available it is possible to determine the form of the probability distribution. The first step is to form a null hypothesis, as to the true form of the distribution. The histogram is very useful in this step, and from observation of Figure 14, it would be natural to consider the hypothesis that the dsitribution is normal. It may be of some benefit to plot the cumulative distribution on normal probability paper; here a straight line is indicative of normality. The distribution below is shown on normal probability paper in Figure 15. Clearly the distribution exhibits the characteristics of normality except at the extremes. At

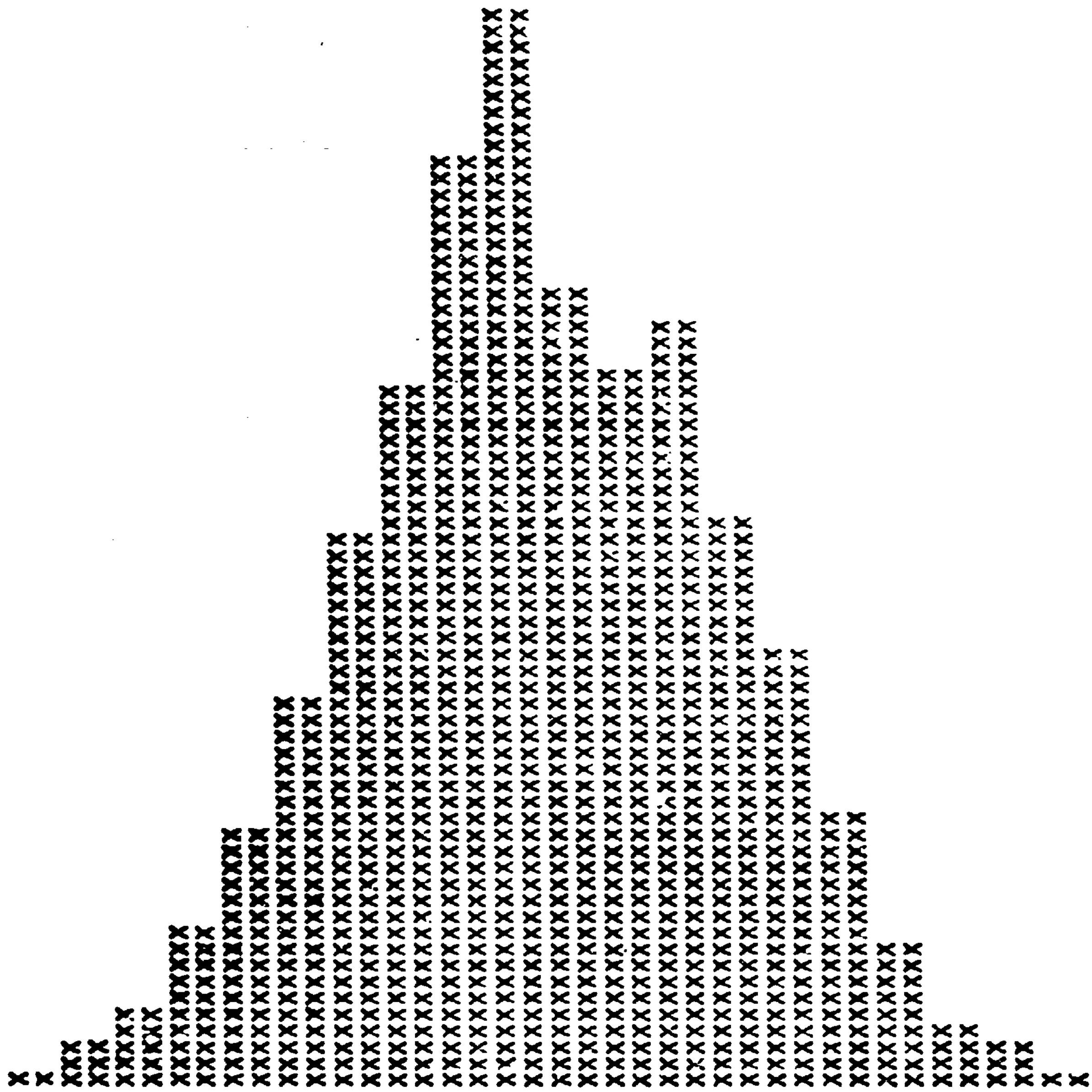

Figure 14. Typical Output Histogram of MLSC's

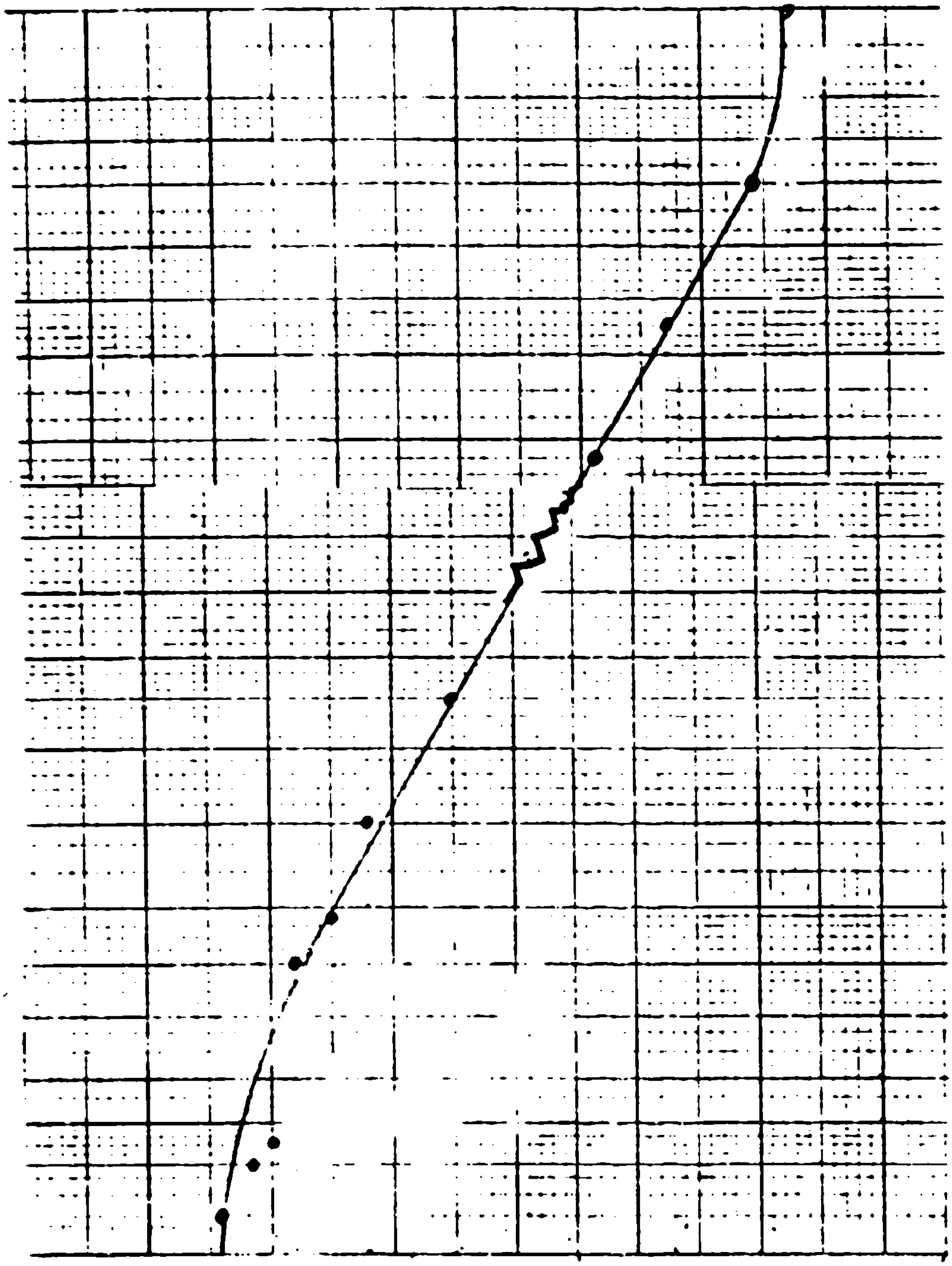

Figure 15. Distribution of MLSC's on Normal Probability Paper (Center section has been deleted).

At approximately 2.5 standard deviations from the mean there is some truncation shown by the curved lines at the top and bottom in Figure 12. The fact that some truncation occurs is not particularly surprising in a sample of 1,000. A sample of 5,000 should eliminate this.

Having formed the null hypothesis that the distribution is normal, it is next necessary to check this assumption statistically. The method used here is the Kolmogorov-Smirnov (K-S) goodness of fit test. In this test, the cumulative distribution function of the empirical data is compared with the cumulative distribution function of the hypothesized distribution. If at any point in the comparison, the difference between the two functions is greater than "D," the K-S statistic, the null hypothesis must be rejected. The value of D is determined by the number of observations in the sample and α, the desired level of risk of rejecting a null hypothesis which is true. In this study an α of .10 and a sample size of 1,000 were used in all tests. This gives a D statistic of .0386. Since the K-S test is more intuitively clear in a graphical presentation, the K-S goodness of fit test for 1,000 samples of MLSC, with MTBF as a random input is shown in Figure 16. Any deviation of more than .0386 between the two depicted lines, is cause for rejection of the null hypothesis. In this case, the null hypothesis is not rejected. In fact, the largest deviation from the theoretical distribution is only .025 which occurs at MLSC = 41.66.

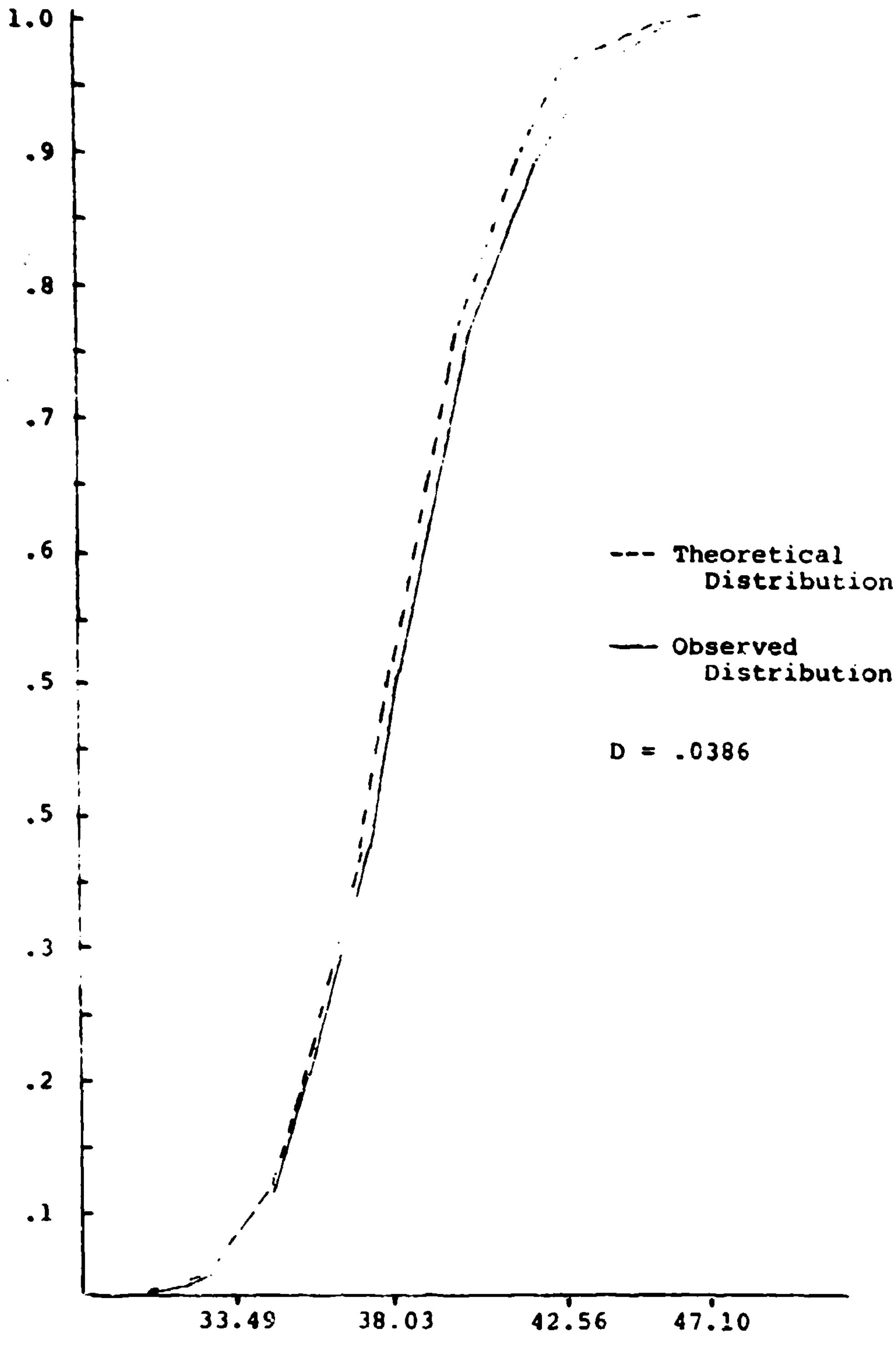

Figure 16. Kolmogorov Smirnov Test for Normality

Interpretation of the Simulation Results

Given that the normality hypothesis is accepted, the probability distribution of MLSC's can now be completely specified. If all input parameters are truly equal to their contractual targets, then the distribution of MLSC's will be a normal distribution with mean $38.4 million and standard deviation $2.8 million. That distribution is depicted in Figure 17.

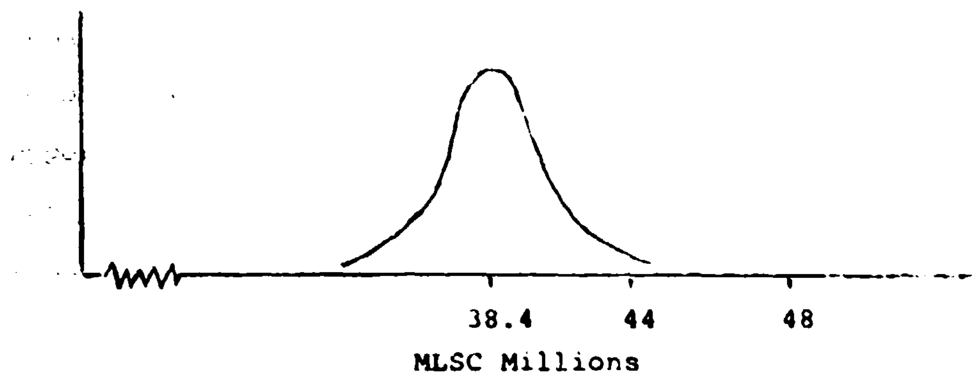

Figure 17. Normal Distribution of MLSC's

Sensitivity Analysis of the Input Distributions

As stated earlier, substitution of a Weibull time to failure distribution, with shape parameter 1.1, has virtually no effect on the output distribution of MLSC's. Substitution of a Weibull time to repair distribution for the lognormal time to repair distribution, however, has a significant effect on the output distribution of MLSC's. The model was run with a Weibull time to repair distribution,

with shape parameter of .18.[5] Under these conditions, the standard deviation of the MLSC is about $12 million. This is about three times the average value of $4 million which occurs when the lognormal time to repair distribution is used. (With all three variables stochastic in both cases). It is apparent that the small shape parameter .18 is causing a very large variance in the time to repair distribution, since the standard deviation of a Weibull distribution is:

$$\sigma_{Weibull} = \sqrt{\theta_1^2\ \Gamma(2/\theta_2+1)-[E(x)]^2}$$

For E(x) = 1.0 the standard deviation is:

$$\sigma_{Weibull} = 22.9$$

This is a very large standard deviation indeed for a mean of 1.0.

The introduction of the above Weibull distribution causes not only a change in the variance of the MLSC distribution, but a change in the form of the distribution as well. The null hypothesis which was actually formed in this thesis was that the distribution was Weibull with scale parameter 43.1 and shape parameter 2.8. These

[5]The shape parameter of .18 is the average value over the seven observations of the equipments which demonstrated Weibull times to repair in the Category II Reliability and Maintainability Tests cited earlier in this thesis (49; 50; 51). The actual values of the seven observations were (.31, .08, .09, .13, .1[illegible] .35, .15).

parameters were estimated using the method of moments and special techniques described in Johnson and Kotz (34).

A Kolmogorov Smirnov (K-S) test conducted at the 90 per cent confidence level causes a rejection of this null hypothesis.[6] Having rejected the hypothesized distribution, it can only be said that the distribution has a Weibull-like appearance and very clearly is not normal.

It is certainly evident then that the validity of the model is dependent upon a correct selection of the time to repair distribution. Nonetheless, the choice of the log-normal distribution in this application is sufficiently well founded in theory and supported by the evidence, that there can be a reasonable level of confidence in the findings.

Making Statistical Inferences Regarding the MLSC Distribution

Returning to the consideration of the normal distribution of MLSC's which results from inputs of exponential time to failure, lognormal time to repair, and binomial fraction reparable this station; it is now possible to make some statistical inferences regarding this distribution. For example, after consulting a table of the normal distribution, it is possible to say that the likelihood of any single random observation from the above distribution being greater than $48 million or 1.25 TLSC is .0003; or,

[6] See Appendix D for The Kolmogorov Smirnov test.

in 10,000 samples, about three observations would be likely to exceed that value. (The value .0003 is for only time to failure stochastic; with all three variables stochastic about 20 observations in 1000 would exceed this value.)

Now suppose all specifications are not met. In particular, suppose that all of the true MTBF's are equal to .9 times their specified value and that all MMH and RTS values are exactly as specified in the contract.[7] This would cause two things to happen to the distribution of Figure 17. Its mean would move to the right, and its standard deviation would increase somewhat. If the MTBF's were now multiplied by .8, the mean would move farther to the right, and the standard deviation would again increase. Multiplying the MTBF's by various factors between 1.1 and .6 would then give a series of probability distributions along the horizontal axis as depicted in Figure 18. Each of these distributions is a normal distribution, but each has a different standard deviation. If the relationship between the mean and standard deviation of these distributions could be determined, then it would be possible to construct a distribution at virtually any point along the axis without repeating the simulation procedure.

[7] The computer model provides a value of FACTOR as a multiplier for input true MTBF.

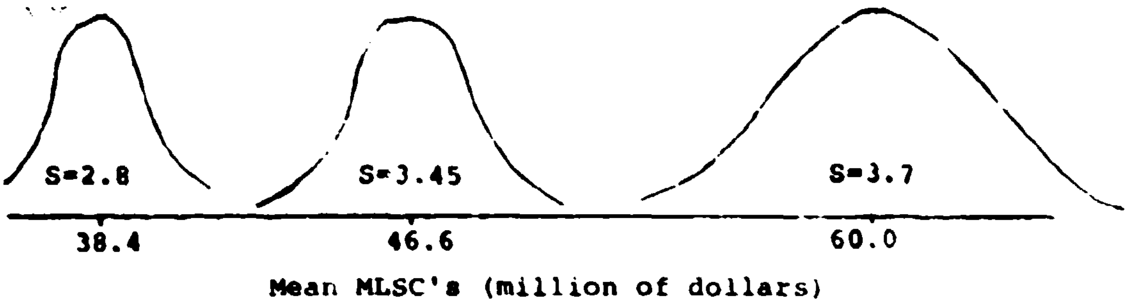

Figure 18. Series of Normal Distributions

In order to determine this relationship, the simulation model was run for 14 values of FACTOR between 1.1 and .6. A regression analysis was then conducted and it was determined that the estimated standard deviation (S) for any value of factor could be found by the following equation[8]:

$$S = 3.98 - 1.17 \cdot \text{FACTOR}$$
$$(33) \quad (8.17) \quad (\text{t statistic})$$

This equation predicts the values quite well for the range of interest. An alternative method for predicting the standard deviation for any given point is to assume that S is a constant percentage of the mean. This method

[8] The regression methodology is outside the scope of this thesis. See (48:375) for a discussion of this topic.

is acceptable over a very short interval, say from factor = 1.1 to factor = .95.[9]

Given that a distribution of MLSC's about any given mean MLSC can be established, then it is possible to make inferences about the results of a test conducted under any specified value of factor. For example, assume the true value of all MTBF's is equal to .75 times the specified value. If 1,000 tests of 3500 flying hours each were conducted under these circumstances, the distribution of MLSC's depicted below would result.

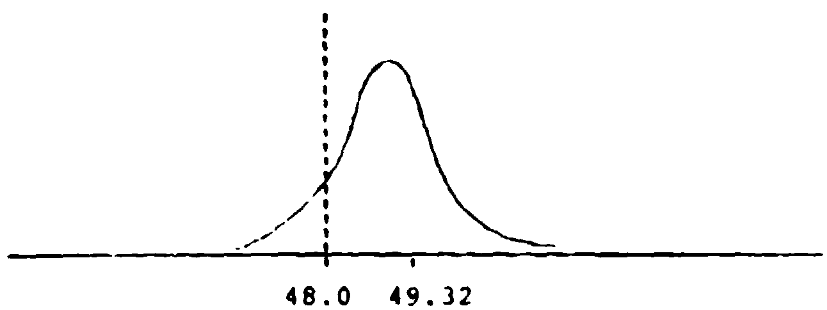

Figure 19. Normal Distribution Mean $49.32

[9] In order to compare the two methods, predictions of 13 values of s were made with each method. These predictions were then compared with the known values. The regression predictions were very good over a range of FACTOR from .6 to 1.1, with the greatest error of prediction being about 5% of the known value. Errors using the percentage of the mean method over this range were as much as 30% of the known values. Over the range of FACTOR from .95 to 1.1, the percentage of the mean predictions were off by no more than 5%.

It can be seen then, that for factor = .75 or equivalantly, true mean LSC = $49.32 million, about 663 observations out of 1,000 would be expected to exceed $48 million.[10] It would also be true that for a single observation, the likelihood of exceeding 48 million is .663. Under these circumstances, only about two observations in 10,000 would be expected to be less than 38.4 million or the probability of a single observation being under 38.4 million is .0002. Translating these statistics into contractual terms; with a true LSC of 49.32 million, if 1,000 tests of 3,500 hours were run, the COD clause would be invoked about 663 times. An award fee would be presented less than once. Stated differently, for a single observation the probability of invoking COD would be .663.

Similar statistics can now be generated for any true mean value of LSC. If these statistics are generated and plotted with LSC on the abscissa and probability on the ordinate, a decision curve such as the one depicted in Figure 20 results.

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[10] It should be noted at this point that the LSC itself is in reality a random variable. That is, if it were possible to measure the actual 15 year support cost over several replications, with all exogenous and endogenous variables equal, then the results would likely be several different observations of LSC. This is consistent with the definition of a random variable, i.e., it is a numerical event whose value will vary in repeated samplings. Thus there can be no "true" value of LSC, but only a true mean value of LSC. In the remainder of this thesis, however, the term true LSC will be used in place of the term, true mean LSC, for the sake of simplicity of exposition.

The two curves on this chart were derived from the two statistics just discussed. The curve on the left represents the probability that an award will be made for any given true value of LSC. More formally stated, the curve is the conditional probability of an award, given a value of true LSC. The curve on the right is simply the probability that a correction of deficiencies will be invoked given a value of true LSC.

To take an example, suppose the true value of LSC is actually 43.6 million where the letter A* appears on Figure 20. From this point a vertical line to intercept the award fee curve shows that when this is the true value of LSC, the probability of an award is .05. Intercepting the right hand curve shows the probability of invoking COD is .17. An equivalent statement would be that, if 1,000 tests of 3,500 hours each were conducted, in about five of these tests an award would be made under the current Support Cost Guarantee provision and in about seven tests the COD provisions would be invoked.

Consider now only the right hand curve from Figure 20. This curve, which has a useful interpretation in the framework of classical statistics, is shown alone in Figure 21. In classical statistical hypothesis testing, this curve represents the power of the test. In particular, the curve of Figure 21 is the power curve with respect to the null hypothesis,

$$Ho: \mu \leq TLSC,$$

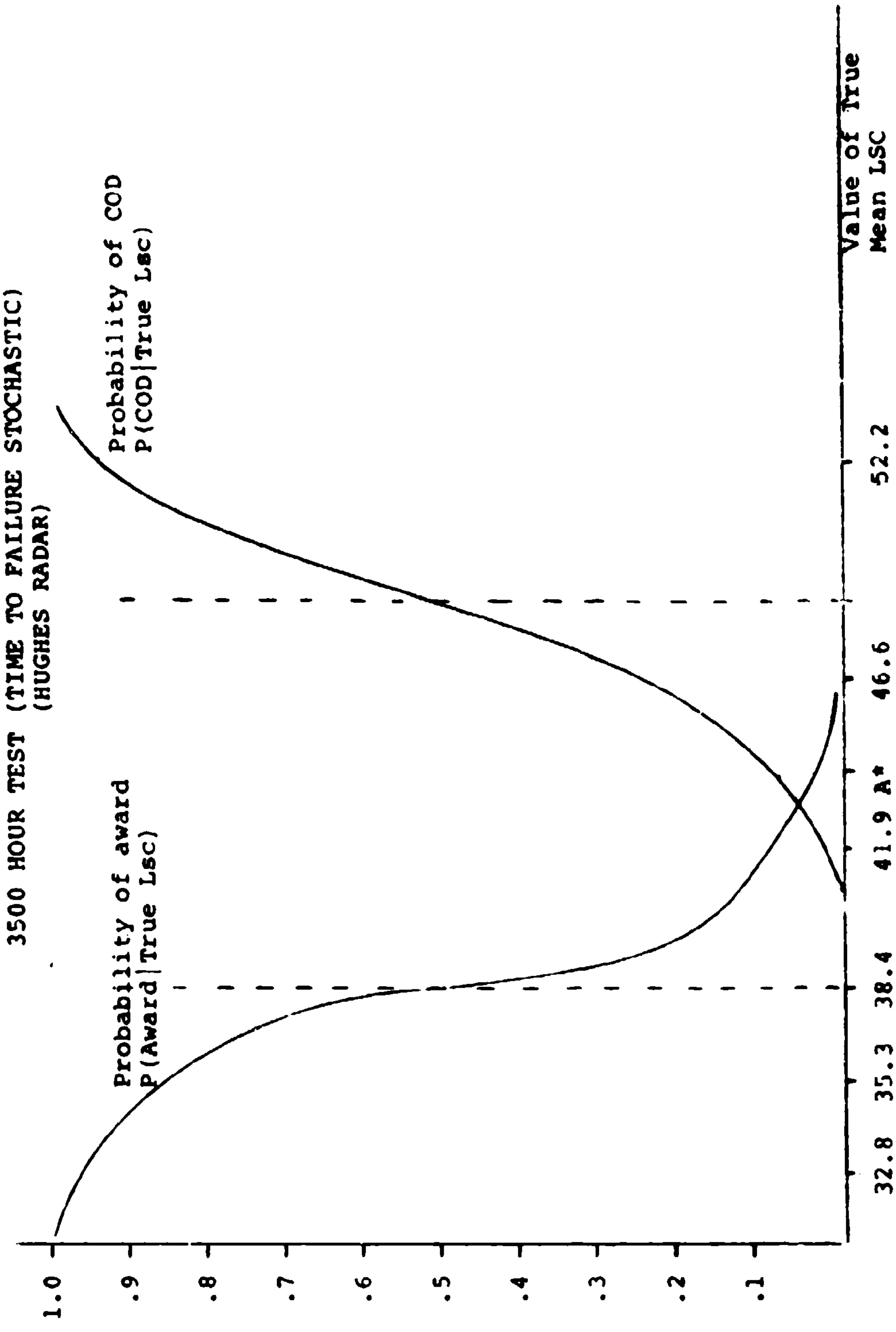

Figure 20. Decision Curve A

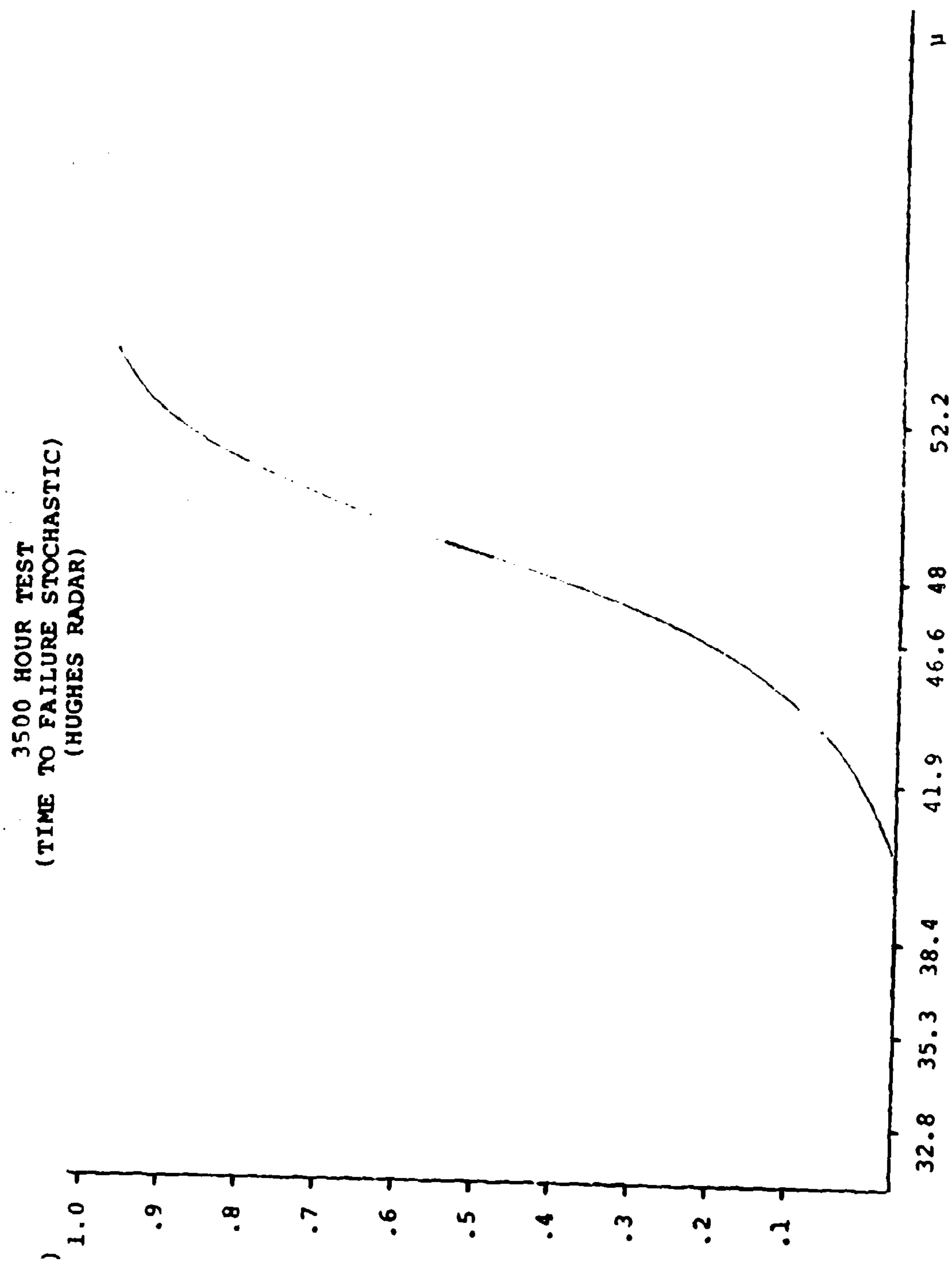

Figure 21. Power of Test Curve

where μ = mean MLSC, versus the alternative hypothesis,

$$Ha: \mu > TLSC.$$

Proceeding from the assumption of normality, the power curve function, $\gamma(\mu)$, can be derived analytically as follows:

$$\gamma(\mu) = \text{Power } (\mu) = 1-\beta\ (\mu)$$

$$= \Pr\ (\text{Accept } Ha|\mu)$$

$$= \Pr\ (MLSC > 1.25\ TLSC|\mu)$$

$$= \Pr\ \left(\frac{MLSC-\mu}{\hat{\sigma}\ (\mu)} > \frac{1.25\ TLSC-\mu}{\hat{\sigma}\ (\mu)}\right), \text{ or}$$

$$\gamma(\mu) = \Pr\ \left(Z > \frac{1.25\ TLSC-\mu}{\hat{\sigma}\ (\mu)}\right),$$

where $\hat{\sigma}\ (\mu)$ is the sample standard deviation as a function of μ.

In classical hypothesis testing, it is the usual practice to specify the desired levels for α, known as the significant of the test, and $\beta\ (\mu_a)$, where μ_a is some specified value of μ. Given these values, the left end point of the critical region (1.25) TLSC in this case) and the sample size (test length in this case) are automatically determined. But in this application, the procedure was reversed, first specifying test length and critical region, and then accepting the resulting values of α and $\beta\ (\mu_a)$.

Given the knowledge of the variance of the MLSC distribution which can be estimated by the methods described in this thesis, a more scientific approach to the statistical

testing of LCC can be visualized. First, it is necessary to develop a general expression for the variance of MLSC in terms of the inputs to C1, C2 and C3. Next, specify α, μ_a and β (μ_a) in the contract. These values would then determine the required test length and the left endpoint of the critical region.

It is interesting to note that the value of α which results from the LSC test described herein is very small, i.e. $\leq .01$.

The Experimental Design

All of the curves shown so far have been those

derived from the LSC model with MTBF as a stochastic input. Now it is appropriate to investigate the effect on the model of assuming that maintenance manhours (MMH) and reparable this station (RTS) are replaced by random variables. The purpose of this part of the investigation is to determine which variables will have a statistically significant effect on the variance of the MLSC. If it can be positively determined that some variable has no significant effect on the variance of the MLSC, then it would probably be reasonable to treat that variable as a deterministic input in this model. Note here that it would have to be shown not only that the variable itself does not have any effect but also that it has no significant interactions with any of the other variables. If such a variable is found, this would mean to the designer of a verification test that this particular variable may be added to or deleted from the model with no significant effect on the amount of uncertainty in the MLSC distribution.

The second purpose of this part of the investigation is to determine the relative importance of each of the random input variables in terms of their contribution to the overall uncertainty in the distribution of MLSC's. The factor, test length is included in the experimental design for the same reason as the other variables, namely to measure its contribution to the uncertainty in the distribution of MLSC's. Of course, it would be expected that the test length would have a negative contribution to the

uncertainty. That is, the variance of the MLSC distribution ought to decrease as test length increases.

This investigation has been carried out using the techniques of experimental design and analysis of variance. A discussion of the methodology involved in that analysis is outside the scope of this thesis. The reader is referred to references (15; 42; 47) for discussions of the techniques.

This investigation was conducted using the analytical framework of a 2^4 complete factorial experiment; meaning there were four separate factors with two levels each. The four factors are: MTBF, MMH, RTS, and verification test length. The levels for the first three are either deterministic or stochastic. For test length the two levels are 3,500 hours and 10,000 hours. The response which is being measured as a function of all of these inputs is the uncertainty (standard deviation) of the MLSC. Each of these factors could be examined separately, but this would not be a true depiction of the behavior of the system, as the interaction among the factors must be considered as well. The analysis of variance techniques account for both the separate and the interacting effects of each factor. Tables VIIIA and VIIIB depict the experimental design and the results of the analysis of variance in the traditional format.

TABLE VIIIA

THE EXPERIMENTAL DESIGN

			Analysis of Variance Observations Two Replications			
			Hughes Radar			
			FACTOR A, MTBF			
			Deterministic		Stochastic	
			FACTOR B, MMH			
FACTOR C, RTS		FACTOR D, TEST LENGTH	Deterministic	Stochastic	Deterministic	Stochastic
	Det.	3.500	0	1.0	2.8	2.7
			0	1.1	2.7	2.85
		10.000	0	.64	1.63	1.69
			0	.60	1.61	1.74
	Stoch.	3.500	3.02	2.6	3.8	3.95
			2.89	3.02	3.87	4.01
		10.000	2.4	1.76	2.39	3.56
			1.67	1.801	2.3	2.39

TABLE VIIIB

ANALYSIS OF VARIANCE

Source	SS	df	MS	F(.10)(1,16) = 4.5
A	1.44	1	1.44	21.3
B	.586	1	.59	8.7
C	1.855	1	1.86	27.4
D	6.24	1	6.24	92.2
AB	.068	1	.02	2.7
AC	1.65	1	1.65	24.4
AD	.666	1	.67	9.8
BC	2.50	1	.25	3.7
BD	.003	1	.003	4.4
CD	.417	1	.42	6.2
ABC	.928	1	.93	13.7
ABD	.149	1	.15	2.2
ACD	.184	1	.18	2.7
BCD	.067	1	.07	.99
ABCD	.01162	1	.01	1.6
SSE	1.08	16	.07	

Table VIIIA shows the observations of variance of MLSC from two full replications of the experiment. The two replications were identical in all respects except for the random number seed. Table VIIIB shows the results of the analysis which was conducted with a standard IBM 360 library routine. In particular the "F" statistic in column 5 of Table VIIIB indicates whether or not the particular effect is significant at the 90 per cent conficence level. In this experiment, a factor is significant if it has an F statistic > 4.5. A total of eight effects and interactions can be said to be statistically significant. All individual factors are significant. The interactions of AC, BD, CD and ABC are significant. The meaning of an interaction between two effects, for example, A and C is that the response given with C at its high level is not independent of the level of A. Or the amount of uncertainty produced by the random variable RTS, depends upon whether MTBF is deterministic or stochastic. The other interactions have similar explanations. Figure 22 graphically depicts the interaction between A and C. Clearly, the amount of change

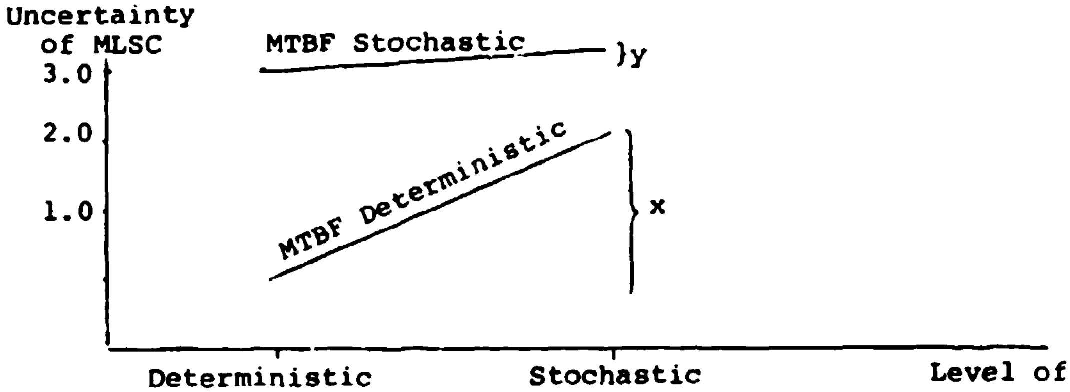

Figure 22. Interaction Between MTBF and RTS.

in the uncertainty with changes in RTS depends on the level of MTBF. With MTBF deterministic a relatively large change "X" occurs. A smaller change "Y" occurs when MTBF is stochastic.

With the significance of the factors and interactions determined, the next step in the analysis is to determine statistically the relative importance of each of the contributions to the uncertainty. This can be accomplished by a technique known as contrasts of factor level means which is discussed in reference (15). This is a comparison of average response averaged over each of the input random variables. The results of this comparison are shown in Table IX.

TABLE IX.

Factor Level Means

Analysis of Standard Deviation of MLSC	
Factor	Mean Response
MTBF Stochastic	2.749
RTS Stochastic	2.839
MMH Stochastic	2.213
90 % Confidence Intervals	
For Difference Between Means	
MTBF - RTS	-.07, .25
MTBF - MMH	.375, .536
RTS - MMH	.466, .787

From the tabied information above, the following deductions

can be made with a joint confidence of 90 per cent:

1. There is no significant difference between the mean response with MTBF stochastic and that with RTS stochastic.

2. The mean response with MTBF stochastic is greater than that with MMH stochastic by .4555 ± .0805.

3. The mean response with RTS stochastic is greater than that with MMH stochastic by .547 ± .0805.

In summary, it can be said that the greatest contributor to the uncertainty of MLSC is either MTBF or RTS; there is no significant difference in their contribution. MMH contributes somewhat less (about 20 per cent less) to the uncertainty than either MTBF or RTS.

This information is useful in understanding the importance of correct measurements. It is no surprise that MTBF is a major contributor to uncertainty of MLSC but the fact that RTS is equally important may not have been intuitively apparent. It is obvious then that measurement errors in MTBF and RTS will contribute significantly to the overall uncertainty. And while measurement errors of MMH are secondary to MTBF and RTS they are only about 20 per cent less important. This information is also useful in planning a simulation model for a given verification test. It should be apparent now that none of the three factors is a negligible contributor to uncertainty, and that all three ought to be included if they are all subject

to verification. If one factor must be omitted, then the omission of MMH would have the smallest impact on the validity of the model.

This discussion of analysis of variance concludes the presentation of the formal methodology.

In the next chapter, several applications of this simulation model will be explored in detail.

Chapter 5

APPLICATIONS

Introduction

In this chapter some applications of the simulation methodology are presented. Three applications will be developed:

1. Analysis of COD invocation ratio and verification test length.

2. Analysis of a contractor strategy.

3. An award fee design.

Application to Determination of Suitable Invocation Ratio and Verification Test Length

The first application to be addressed is determination of a suitable COD invocation ratio and verification test length for a given contract. The first step in such an analysis is to establish a conditional distribution of MLSC's at any point along the MLSC continuum. This is most easily accomplished by manipulating the input data to produce a given mean MLSC, and then examining the variance and type of distribution at each of say ten points. If it can be seen that the type of distribution is invariant, and that the variance changes in a linear fashion, then the variance for any point on the scale can be found by conducting a linear regression on the ten existing data

points. A second method of predicting the variance at any point on the scale is to assume a constant mean to variance ratio. This method has been found to be less accurate than the linear regression, however.

If there is a need to investigate test length, then that variable must be introduced at this point. The conditional distributions of MLSC must be established over the range of test lengths of interest.

Next the decision curves can be constructed. A decision curve is defined in general here as a curve which shows the conditional probability that action X should be taken, that is, the probability that action X should be taken, given an observation of MLSC. Recall that in general such curves may be constructed for more than one observation of MLSC. Figure 23 depicts the decision curves for the F-16 study as a function of test lengths from 1,000 to 10,000 hours. If the problem is just to determine what degree of uncertainty exists for a decision based on an observation of MLSC for various test lengths then the task is complete.

If the problem of interest is a more fundamental one of, for example, determining what the decision criterion should be then some additional analysis is required. Consider the case of the F-16 invocation ratio. Suppose for a moment that this decision (the COD invocation ratio) is still to be made; and further suppose that the verification

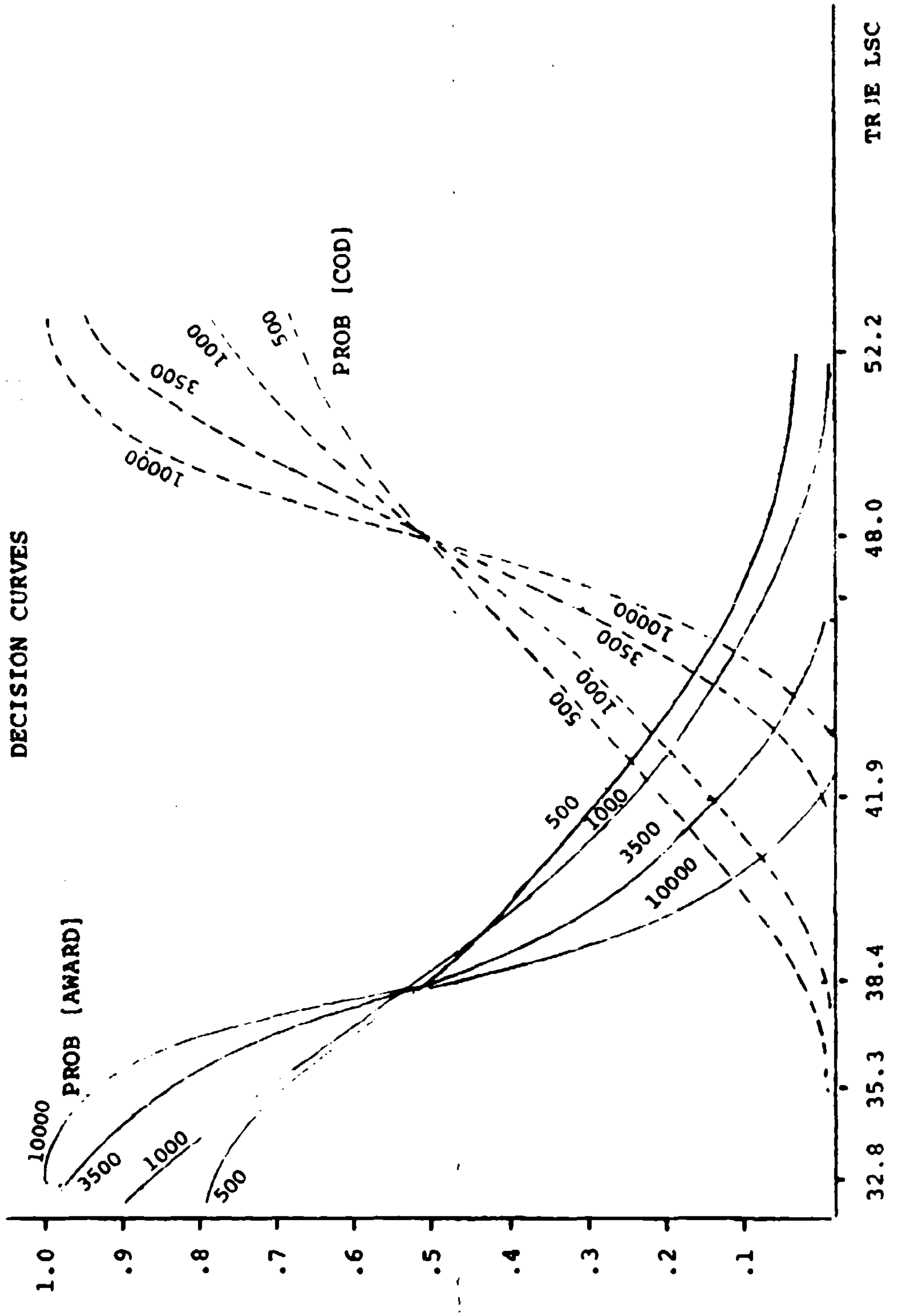

Figure 23. Decision Curves as a Function of Test Length

test length may be varied. Suppose that, there was, a proposal for a COD invocation ratio of 1.1, or ten per cent above TLSC. In this example assume that the contractor indicated a desire for a higher invocation ratio and a willingness to reduce the price of the COD guarantee at a rate of $267,000 per percentage point to an invocation ratio of 1.25. The contractor is implicitly stating that the statistical risk of erronaous invocation, which will be called Beta, is too great at the 1.1 ratio. In this situation, Beta is found to be about 17 per cent. The Beta for the invocation ratio of 1.25 is found to be slightly less than 1.0 per cent. An imputed value can now be placed on each percentage point of risk in the perception of the contractor. Since the contractor is willing to pay $4,005,000 for a 16 per cent reduction in risk, it is apparent that the imputed value of that risk reduction is approximately $250,000 per percentage point of Beta.

Suppose now that the level of risk desired by the contractor can be provided by adjusting the test length as well as the invocation ratio. And further suppose that the government has a desire to hold the invocation ratio at 1.1. Then the requirements of both parties can in theory be satisfied in two ways: 1. Pay the contractor the price of increased risk. 2. Reduce the risk through increased test length (and absorb the additional testing costs.)

From a chart similar to Figure 23 it can be determined that a 10,000 hour test would reduce the Beta to about 9.8 per cent for an invocation ratio of 1.1. Assuming a linear relationship then it can be said that each 900 hours of test time decreases Beta by one per cent. Extrapolating beyond 10,000 hours, it can be determined that a Beta of one per cent will be attained at 17900 hours of testing. Certainly, this is an extraordinary test length and it is unlikely such a lengthy test would be undertaken. For more reasonable levels of Beta, say 10 to 15 per cent, much more reasonable test lengths would be expected.[1]

To carry the above analysis one step further, it would be possible in this situation for the government to compare the cost of buying the COD guarantee at the 3,500 hour Beta level with the cost of testing to reduce the Beta to one per cent. The contractor has already (implicitly) stated that each one per cent reduction in Beta will bring a $250,300 reduction in COD guarantee price. Equating 900 hours per percent to $250,300 per percent it is apparent that if the price of testing is less than $278 per flying hour, then it would be less expensive to increase test length than to pay for the higher Beta risk.

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[1]An interpolation or extrapolation such as demonstrated here should be undertaken with some caution, as the assumption of linearity is questionable.

Use of the Model for Analysis of Contractor Strategy

The simulation methodology developed here can be used quite conveniently in evaluating the feasibility of various contractor strategies both before and after the fact of contract negotiation. The example presented here is after the fact.

Suppose in the current contract situation a significant underrun of MTBF's were to occur. In particular, suppose the MTBF's were only slightly better than those of the previous or baseline equipment upon which the forecasts were based. This will be referred to as a condition of "minimum technological improvement." The values of MTBF to be used for the minimum technological improvement were decided in a conference with Mr. Perry Stewart (2). The values used are shown in Table X along with the originally predicted values of MTBF.

TABLE X

MINIMUM TECHNOLOGICAL IMPROVEMENT SCENARIO

FLU	Minimum Technological Improvement MTBF	Original MTBF
HUD	100	172
Navigation Unit	110	200
Fire Control Computer	275	428
Electronics, HUD	215	385
Flight Control Computer	144	144
Radar EO Display	80	188
Digital Scan Converter	150	274
Electronics EO	188	188
Hughes Radar Components		
Received/Exciter	220	340
Data Processor	180	274
Signal Processor	180	282
Transmitter	100	144
Antenna	160	315

The average MTBF predicted under the minimum technological improvement scenario is about .6 of that which is predicted in the contract. After considering the historical experiences in MTBF predictions discussed in Chapter 1, this scenario does not seem to be one which is totally outside the realm of possibility. Consider, for example, the latest AFM 66-1 data on field MTBF for the baseline items for the first two FLUs above. The A7D HUD is demonstrating a field MTBF of 38 hours. The inertial navigation unit, a field MTBF of 45 hours (3).

Assume then, that those MTBFs which the equipment appears to be capable of delivering at the time of initiation of the 3,500 hour verification test are as shown in Table X. Given these conditions, what is the contractor's ability to control the outcome of the logistics supportability evaluation tests? This question assumes as well that the contractor is able to estimate those approximate values of MTBF which will be delivered.

There is, of course, only one input which is subject to verification which can be directly manipulated by the contractor without changes in hardware. That is the unit cost of each FLU.[2] The question then, is how much leverage can be exercised through control of FLU unit cost to correct

[2] The term Unit Cost (UC) which will be used in the cost equations for the verification test refers to the average unit cost of the FLU in effect during the period of the verification test.

for high logistics support costs which are resulting from low MTBF's.

Under the conditions described above, with the true MTBF's equal to those described under minimum technological improvement and no other changes to any inputs, the mean of the distribution of measured logistic support costs would be $61.6 million. The likelihood that one observation of MLSC would result in an invocation of COD under these conditions is greater than .99. Suppose the contractor could afford to reduce the average unit prices of each FLU to say 70 per cent of the original predicted price. This would result in a mean MLSC of 48.0 million, and a likelihood of invoking COD of about .5.

Now suppose the contractor desired a small risk of invoking COD, say on the order of .10. This amount of risk could be established by reducing unit cost per FLU to .625 of the predicted value.[3]

Although it seems somewhat unlikely that award fees would be a matter for consideration in this contractor strategy, it is interesting to note that if the unit cost of each FLU is reduced to .50 of the predicted value, then the likelihood of receiving an award is .38 while the likelihood of having COD invoked is virtually zero. If the

[3]These values were derived by uniformly reducing the cost per FLU by the given factor. The resulting probability distribution of MLSC's was then analyzed in order to derive the conditional probability of these events.

entire award were to be given for an MLSC $\leq$ TLSC then the expected value of the award for this (average unit cost equal .5) position would be \$760,000. (.38 x 2.0 million).

As a result of this brief investigation of a contractor strategy two statements can be made.

1. The simulation methodology developed here is a convenience tool for analysis of contractor strategy.

2. In the case at hand, it can be seen that the F-16 contractor can exercise considerable control over the outcome of the LSC test, even under a minimum technological improvement scenario. In fact, the contractor can, by manipulating unit cost, establish virtually any desired level of risk of invocation of COD. It is beyond the scope of this thesis to investigate the relative economic advantage to the contractor of this kind of strategy. It is worth considering though, that such a strategy of avoiding COD might result more from a desire to prevent damage to the contractor's reputation and future potential than from pure short run economic considerations (54:321).

Award Fee Designs

In this section, the simulation methodology will be exercised in formulating an award fee design. The particular award fee design presented in this analysis will be applicable to the F-16 control FLU award fee in the amount of \$2,000,000. The methodology, or some variation thereof, can readily be adapted to employment in other similar

award fee applications. The fundamental prerequisite to the application of this methodology is that the amount of the award fee be unilaterally determined by the government based only upon the attainment of some measurable logistics supportability goal.

Considerations in Formulating a Design

There are two general statements which can probably be made about formulation of any award fee plans based on logistics supportability.

First, any plan must be objective enough to avoid even the appearance of arbitrary or capricious determination of the fee.

Second, the major consideration in evaluation for the award fee must be the contractor's performance in minimizing logistic support cost.

A third consideration ought to apply in cases where there is statistical uncertainty involved in the measurement of goal attainment. This is, simply that the uncertainties ought to be considered in the determination of the fee.

With this last point in mind, the methodology described here will construct an award fee curve which is based on the "amount of confidence" that an award is truly deserved. The "amount of confidence" factor will be applied to a somewhat artificial sharing ratio which will in turn determine the absolute amount of the award

for any given value of MLSC.

Precedent

There is no exact precedent for this award, but there have been applications which bear some resemblance to this. In particular, the ARC-XXX/ARC-164 program employed an incentive/penalty contract which was based on measured logistic supportability. The ARC-164 program actually established the incentive as an acquisition cost adjustment, but the application is still sufficiently analagous to be of interest here. For the ARC-164 the following award arrangement was established:

Where: MLCC is measured life cycle cost:

And TLCC is target life cycle cost

If $|MLCC-TLCC| \leq .03$ TLCC; no award

If MLCC $\leq .97$ TLCC then;

$$\text{Award} = .5\ (.97\ \text{TLCC}-\text{MLCC})$$

Stated verbally, a three per cent "dead spot" was established to allow for statistical uncertainty. Beyond that point a 50/50 sharing ratio was established, with the contractor receiving 50 per cent on every dollar that he was able to reduce MLCC up to the maximum potential award fee (53:15).

The Methodology

The methodology suggested here incorporated the ideas above and one additional concept. The additional

concept is that some non zero amount of award ought to be made if the measured logistic support cost is equal to the target logistic support cost. In other words, there is no "dead spot" in this design.

Using only three premises then, it is possible to construct a reasonable award curve. The premises are:

1. Maximum sharing ratio is 50 per cent.

2. Dollar amount of award increases with the probability that LSC is truly less than TLSC.

3. Some positive dollar amount of award is made at MLSC = TLSC.

The generalized award fee design depicted in Figure 24 was constructed using these three rules.

The following procedures were used in actually formulating the design.

The maximum award is given at that point where MLSC is two standard deviations, 2σ, below TLSC. This point, 2σ below TLSC, is the point where it is virtually certain that true LSC is below target LSC. That is, it is that value of an observation which would result in a 98 per cent left-hand, one-sided confidence interval whose right end point is $\leq$ TLSC. To see this, note that

$$P\ (\mu < TLSC) = .98$$

$$= P\ \left(\frac{-MLSC + \mu}{\sigma(MLSC)} < \frac{TLSC - MLSC}{\sigma(MLSC)}\right)$$

$$= P\ \left(\frac{MLSC - \mu}{\sigma(MLSC)} > \frac{MLSC - TLSC}{\sigma(MLSC)}\right)$$

Since $\frac{MLSC-\mu}{\sigma(MLSC)}$ is a standard normal deviate, Z, we have

$$P\left(Z > \frac{MLSC - TLSC}{\sigma(MLSC)}\right) = .98,$$

which suggests that

$$-\left(\frac{MLSC^* - TLSC}{\sigma MLSC}\right) = Z_{.02} \approx 2.0$$

so that

$$TLSC - MLSC^* = 2.0\ \sigma(MLSC)$$

or

$$MLSC^* = TLSC - 2.0\ \sigma(MLSC).$$

The maximum sharing ratio of 50/50 is maintained to a point which is 1.5 standard deviations below TLSC. Over this entire range, from 2σ to 1.5σ the average probability that true LSC is below TLSC is approximately .96.

Over the range of MLSC's between 1.0σ and 1.5σ below target the sharing ratio is reduced to 70/30 (Government/contractor). Over this range the average probability is .89.

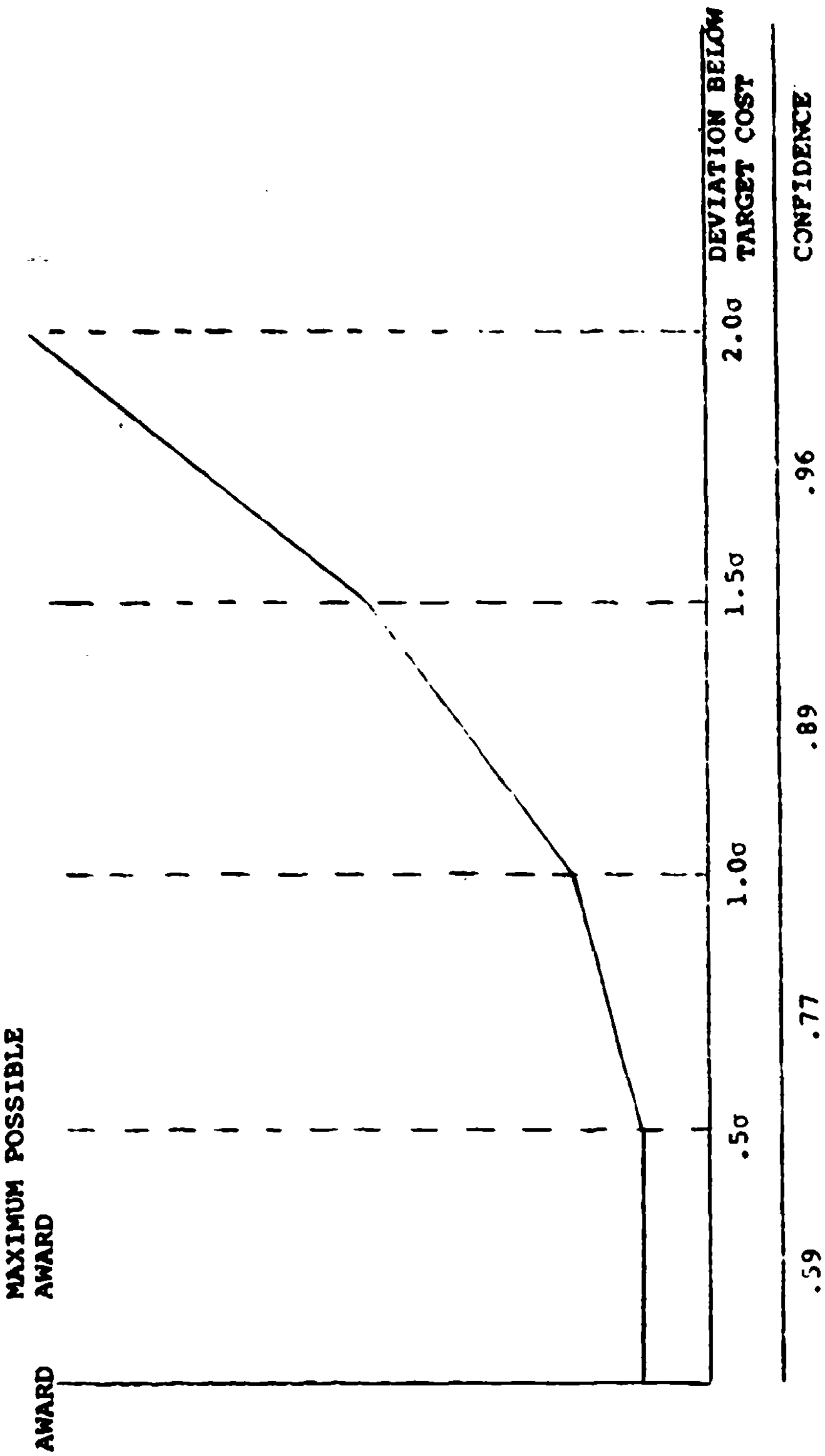

Figure 24. Award Fee Design General

Over the range of MLSC's which are between .5σ and 1.0σ below target the sharing ratio is reduced again to 90/10. Over this range the average probability that MLSC is less than TLSC is .77.

Finally then over the range of MLSC's between 0.0σ and .5σ the sharing ratio is reduced to zero. Hence, starting at MLSC=TLSC, motivation to reduce MLSC increases as MLSC gets smaller since a greater fraction of LSC savings goes to the contractor.

In order to adapt this design to some situations, it may be necessary to adjust the sharing ratios of the two center quarters depending upon the relative magnitude of the award and the logistic support cost being measured. In some cases it might be desirable to use a division of other than one half a standard deviation.

Figure 25 shows the application of this award fee design to the F-16 control FLUs. The standard deviation used here was for all three inputs stochastic, time to failure, manhours, and fraction repaired this station. If only MTBF were to be considered stochastic then the maximum award would occur at a higher value of MLSC.

It is surely apparent that such a dogmatic scheme as this if accepted would leave little room for interpretation on the part of the fee determining official. Certainly such a plan can be expected to provide nothing more than a well reasoned guide to bolster the intuition.

If the curve is to be adjusted before measurement of

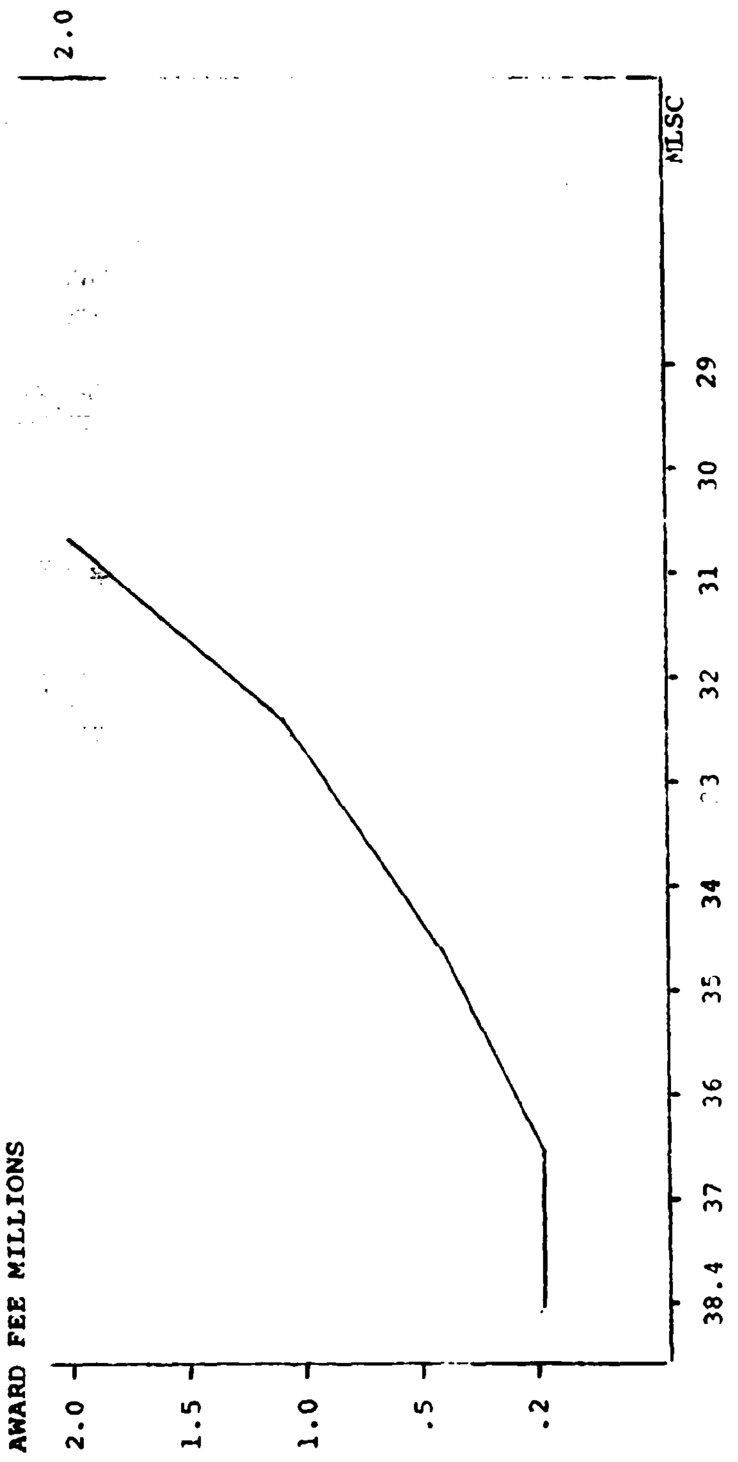

Figure 25. Award Fee Design F-16.

LSC, to account for subjective considerations, then it would be reasonable to move the entire curve in a vertical (rather than horizontal) manner. This would provide an adjustment in the base amount of the award fee for MLSC = TLSC and an equal adjustment for any given observation of MLSC. Of course, an increase in the base award will provide for the full award to be given at a higher value of MLSC. For example, if the base award is raised to $1,000,000 the maximum award will occur at 32.2 million MLSC or at approximately 1.6 standard deviations below the target logistic support cost. Such a point (1.6σ below the target) would still provide a confidence level of about 94 per cent that the true LSC is below TLSC.

Limitations

The award fee design described above is a product of all of the assumptions of both the logistic support cost model and the simulation methodology described herein. It is most directly dependent upon the probability distribution of MLSC. If an error is made in specifying the form of the distribution of MLSC's, considerable error would be introduced into the methodology. It has been determined for example, that if the time to repair distributions have a Weibull distribution with a small shape parameter, say .18, then the distribution of MLSC's will have a far greater variance than under other distributions. It has also been determined that the distribution of MLSC's, with

a Weibull time to repair distribution, will have a highly skewed Weibull-type distribution. If then, in some application of this methodology, it were determined that the manhour inputs had a Weibull distribution, it could be expected that the distribution of MLSC's would be non-normal.

It should also be noted here that in any application of this award fee methodology, the actual amount of award received will be dependent on the verification test length. Since an increase in test length reduces the standard deviation of the MLSC, for any given value of MLSC $\leq$ TLSC, a greater award will be given for a greater test length. This fact might increase contractor motivation to seek greater test lengths.

As a final point it should be noted once again that the logistic support cost used here is not inclusive of all support costs. Rather, it is as described in Chapter 1, a measurable standard against which to compare relative performance. This fact calls into question the concept of a sharing ratio. The question is: is it meaningful to establish a sharing ratio in terms of a measurement which does not include all relevant costs? In fact, it can also be argued that the measurement includes some costs which are not relevant; for example, some would argue that maintenance manhour costs are not relevant. In support of the methodology used here then, it can be said that; even acknowledging the weaknesses of the data, the measured

logistic support cost determined in the 3,500 hour test is the only verified logistic support cost in existence. In this light it seems appropriate to establish a sharing ratio based on this known cost.

The three applications presented in this chapter should give the reader a general idea of the utility of the methodology. Certainly the usefulness of the model is not limited to these particular applications. To facilitate the use of this model in other investigations, a generalization of the methodology is included in the final chapter along with the conclusions and recommendations for further study.

Chapter 6

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS FOR FURTHER STUDY

Introduction

This chapter is presented as a summary and as a generalization of the methodology so that it might readily be adapted by the reader to other applications of logistic supportability measurement and evaluation. Also included here are the conclusions and recommendations for further study.

Data

Data collection for input to the simulation model may be a relatively simple matter if the applicable contract has been drawn up and if such data as MTBF, MMH, RTS, RIP, etc. are contractually specified. The credibility of the contractual specifications can only be evaluated on a case by case basis, and for those data elements which are subject to verification, the credibility will evantually be determined by testing. Data collection, or more correctly, data forecasting for systems in early development phase must be based on AFM 66-1 data from similar systems.[1]

[1] For a discussion of data collection problems and some new data collection systems see Estimation of Life Cycle Costs: A Case Study of the A7D by Marco Fiorello (23).

The LSC Model, Choice of Terms

In tailoring the LSC model to a specific application, it must be realized that in some cases inclusion of fewer terms may provide greater accuracy. Certainly, this would be true when the data inputs for some of the terms are particularly weak. The decision as to which terms of the LSC model to employ can only be made on a case by case basis after consideration of the relative importance of each term to the decision at hand and the ease and accuracy with which it can be verified.

Use of Monte Carlo Methods or Analytical Methods

Analytical methods may be feasible in some applications. If, for example, it is necessary to examine the variance of MLSC as a function of only one input random variable, then it may be possible to use propagation of errors techniques. It is unlikely that convolution integrals would be useable in analyzing anything so complex as a life cycle cost model. In considering the use of propagation of errors techniques, it is important to recall that each of the input distributions must be normal.

The Monte Carlo Method

In subsequent discussion, it will be assumed that the Monte Carlo method is being employed; much of the development would follow a precisely parallel path in analytical methodology.

After decisions have been made as to which of the LSC model parameters should be subject to verification; and among this group which should be treated as random variables, then the next step is to select probability distributions for these random variables. As has been shown, theory is a powerful tool in making this selection when faced with a paucity of relevant data. Data are available on distributions of time to repair and time to failure on certain equipments. These data have been collected and fitted to theoretical distributions during Category II reliability and maintainability testing at the AF Flight Test Center. In using these data, caution should be exercised to attempt to find equipments which are similar in function and construction.

If an actual empirical distribution on a particular equipment is available, it can be incorporated into the simulation model. In the usual case, however, it will be necessary to employ a theoretical distribution. Conversion of the theoretical distribution into a Monte Carlo process generator is a relatively simple matter as illustrated in the appendix for exponential, Weibull, and lognormal distributions.

In some cases, the determination of the correct probability distribution may not be very clear. In such situations it is advisable to incorporate the "second best" distribution into the model in order to determine whether the difference in inputs will have a significant impact

on the output. As demonstrated in this study, the incorporation of a Weibull distribution of failures (with shape parameter 1.1) caused an insignificant change in output. The incorporation of a Weibull distribution of time to repair (with shape parameter .18) however, caused an extraordinary change in output variation. The lessons learned from this sensitivity analysis are:

1. It is not necessary to be particularly concerned about whether the time to failure is more accurately represented by the exponential or Weibull distribution.

2. It is important to determine as conclusively as possible whether the time to repair of a given item should be lognormal or Weibull. If sufficient data were available, consideration could be given to representing each FLU by its own appropriate distribution.

Before leaving the subject of sensitivity analysis, the subject of experimental design should be discussed, since the experimental design in this context is just a highly formalized sensitivity analysis. When more than two factors are involved in a sensitivity analysis, intuitive methods fail and the framework of experimental design is necessary. In the case at hand four factors were involved (MTBF, MMH, RTS, and test length). Each factor had two levels. Also, interactions were expected among certain of the terms. Virtually any number of factors with up to three factor levels each can be handled by the methods of references 15, 42, and 47.

With the form of the input parameters determined and incorporated, the next step is the determination of the form and parameters of the output distribution (distribution of MLSC's). Three items of information are indispensable in this determination and ought to be included as a part of the computer print out. These items are the mean and standard deviation of the distribution and a histogram. The histogram is essential in forming an educated null hypothesis as to what the distribution appears to be. The mean and standard deviation are required to form estimators of the hypothesized distribution's parameters.

The Kolmogorov-Smirnov (K-S) test can be used at this point in accepting or rejecting the distribution specified in the null hypothesis. If the hypothesized distribution is non-normal, its parameters may have to be estimated by the method of moments (42:300). If the hypothesized distribution is normal, then of course the computer output of mean and standard deviation are the estimators of the parameters. For a normal distribution the K-S test may be conducted using the normal tables. For other distributions, it may be necessary to find the points of the cumulative density function by:

$$F(Y) = \int_{-\infty}^{Y} f\ (t)\ dt$$

where f(t) is the hypothesized probability density function.

The last step in the generalized methodology is determination of the form of the probability distribution

of MLSC's. The next step in the analysis will be dependent upon the particular application.

Conclusions

One of the stimuli for this thesis was a study by the RAND Corporation which questioned the deterministic use of the AFLC Logistics Support Cost Model as a tool for acquisition decision making (62). The hypothesis suggested by that study was that the AFLC Logistic Support Cost Model might be of greater utility in a stochastic form. This study definitely supports that hypothesis. It has been shown in this study that a knowledge of the statistical variance of the Measured Logistic Support Cost can be useful in making decisions about contractual incentives and verification test length. It can be seen that the uncertainty (standard deviation) of the Measured Logistic Support Cost is not so large as to create unacceptable uncertainties in most situations.

Depending upon what assumptions are made, it would be possible to state that the standard deviation of the MLSC is between 7% and 29% of the mean. The 7% figure is for 13 FLU's with only MTBF stochastic, while the 29% figure is for one FLU with three stochastic inputs; MTBF, MMH and RTS. A meaningful average figure might be 19% of the mean which is for 13 FLU's with three stochastic inputs. This figure, (19% of the mean) might be called representative because it is likely that in future tests of aircraft

logistic support cost there will be some relatively small group of control FLU's. It is likely that verification will be required for at least MTBF, MMH, and RTS, thus requiring stochastic representation of these inputs in the model. It can be seen from Appendix A that the standard deviation to mean ratio varies with the number of FLU's tested, at least over the range between one and 25 FLU's. It would be of interest in further studies to determine the behavior of this ratio over a greater range.

The hypothesis stated in the first chapter of this study; namely that the major contributors to uncertainty would be, in descending order of importance, MTBF, MMH, and RTS has been disproven. In fact, the study indicates that the contributions to uncertainty of MTBF and RTS are about equal while the contribution of MMH is secondary.

The practical benefits of this study have been shown in the three different applications demonstrated herein. These are; determination of suitable test length and COD invocation ratios, contractor strategy investigation, and award fee design. The methodology is useful as well in any investigation which involves prediction and verification of Life Cycle Cost.

Recommendations

An area which appears to be a potentially fruitful one for further research into Life Cycle Cost Prediction and Verification, is that of test length optimization.

is an area where it is likely the simulation methodology shown here could be profitably used.

In particular, if the benefits of increased test lengths could be compared with the costs of increased testing, then, in theory at least, a test length could be found which would minimize the total cost of testing. The first problem which arises here is prediction of the variable costs of testing. One method of estimating these costs is a parametric Cost Estimating Relationship (CER) published in a previous study (41:9-2)

Here the cost of the LSC analysis in Dollars is:

$$C = (500)(A)(N^{.93})$$

where N is the number of FLU's, and A is an experience factor with a mean value of .85.

This equation reportedly predicts the cost of performing a typical support cost analysis, that is, testing and collection of the data required to input to the Logistic Support Cost Model. Two problems arise in attempting to apply this CER in this application. First, the definition of typical is not given. Without some way of determining what is the typical test length considered is this CER, it is impossible to use it in estimating the costs of testing over various fixed length tests.

The second problem is that the equation predicts such small values for testing cost as to be simply not believable. For example, assume a very small value of typical test length, say 500 hours. Based on this

assumption, the equation would predict a cost of about $32.000 for testing 13 FLU's over 3500 hours. This seems an extremely low estimate.

Assuming, though, that a credible prediction of the variable costs of testing can be made by a CER or other methods, then some hourly cost can be assigned for LSC testing.

Determination of the benefits of testing is somewhat more complex. The benefits must be measured in terms of decision costs. That is; how much improvement in decision reliability results from each hour of increased testing? An increase in decision reliability then can be described as a decrease in the costs associated with bad decisions. Now, what are the costs of bad decisions? In a contract which involves a COD clause, one of the most important bad decision costs would be an erroneous invocation of COD. This cost would, of course, decrease as test length increases. Another bad decision cost would be the cost of not invoking COD when it should be invoked. This cost would also decrease as test length increases. This concept can be expressed symbolically as:

$$\text{(6-1)} \quad \text{Bad Decision Cost (BDC)} = \int_{-\infty}^{\infty} \begin{pmatrix}\text{Cost of Incorrect} \\ \text{decision if MLSC=X}\end{pmatrix} f(x)\,dx$$

where x is a value of Measured Logistic Support Cost and f(x) is the PDF of MLSC.

It is only meaningful to discuss these erroneous decision costs however, in terms of some true value of LSC, for

without some true value of LSC it is not possible to say what action is correct or incorrect.

Assume then for purposes of illustration that the true value of LSC is equal to TLSC. What then are the bad decision costs, and how do they vary with test length? If true LSC equals TLSC there is only one incorrect decision which can be made; That is an erroneous invocation of COD. Symbolically, the Bad Decision Costs are:

$$(6\text{-}2) \quad BDC = \int_{-\infty}^{1.25\ TLSC} \begin{Bmatrix} \text{Cost of failing to invoke} \\ \text{COD when it should be invoked} \end{Bmatrix} f(x)\,dx + \int_{1.25\ TLSC}^{\infty} \begin{Bmatrix} \text{Cost of Erroneous} \\ \text{Invocation of COD} \end{Bmatrix} f(x)\,dx$$

The first term in (6-2) above is zero if true LSC=TLSC. The second term can be estimated by the following algorithm.

First, assume True LSC=TLSC.

Also, assume that COD will be invoked whenever a single observation of MLSC is greater than 1.25 times TLSC.

Assume that, among the 13 FLU's on the above observation, COD action will be taken on whichever FLU's demonstrate an MTBF less than .75 times the contract specified MTBF. Assume there are j FLU's in this category.

Given the above assumptions, then the costs to the government of COD may be estimated through a method developed by Balaban (5:93). Using this method, if a modification is made to reduce the failure rate, λ, it is assumed that $\lambda_{new}=M\lambda$ where M is a function of both

the actual and the specified failure rate, λ^*. Also M has some minimum value M' so that no modification can reduce λ to zero. M then is defined:

(6-3)
$$M = M' + (1-M')\left(\frac{M^*-M'}{1-M'}\right)^{\lambda^*/\lambda}$$

where M^* is the improvement factor expected at $\lambda=\lambda^*$. Equation 6-3 must be iteratively applied to each of the j FLU's until each of them is modified to its specified failure rate. Assume there are k modifications required for the jth FLU. The cost of each modification is found by: Cost of Modification, $C(M)=1.06(e^{(1-M)/10M}-1)C_p$ [2] where C_p is the unit cost of the FLU. The total cost of modification can now be found as:

$$\sum_{j=1}^{n} \sum_{l=1}^{k} C(M)_{jl}$$

A caution must be included here. Balaban warns that this model has been developed using very limited data and is included for illustrative purposes. Its credibility, therefore is limited.

Accepting this method for the time being however, as the only available estimator, it can be seen that the integral in 6-1 can be evaluated. Realizing that the variance of MLSC will change with test length, this

[2] C(M) is the total cost of modification. It should be multiplied by an appropriate factor representing the government share of modification costs.

integral should be evaluated for several values of test length. Hopefully then a curve something like Figure 26 could be constructed. This would represent the sum of the variable testing costs and Bad Decision Costs.

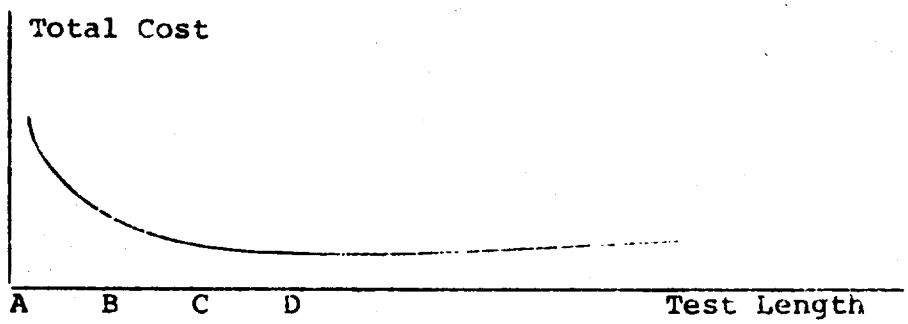

Figure 26. Total Cost by Test Length
True LSC=TLSC (hypothetical).

If the curve has a minimum at a reasonable test length, then this minimum would represent the optimum test length. If the minimum occurs at a value of test length which is too large to be acceptable, then a point on the curve should be chosen where the marginal return is becoming very small, say point B in Figure 26.

The above procedure would derive a curve based upon the assumption that True LSC equals TLSC. It may be of interest to find a curve for an assumption of True LSC equals 1.5 times TLSC. Here the second integral in 6-2 would be zero and the first integral would determine relevant costs. That is:

$$BDC = \int_{-\infty}^{1.25\,TLSC} ((1.5TLSC-(TLSC+\text{Cost of Modifications}))f(x)dx$$

Bad decision costs here are the difference between the True

LSC and the TLSC less the government cost of modification. The integrals in both the situations described above would be evaluated through the simulation methodology. Evaluating the Bad Decision Costs for several values of True LSC might give a series of curves as depicted in Figure 27.

Figure 27. Total Cost by Test Length for Several Assumed Values of True LSC (Hypothetical).

These curves then, hopefully could be evaluated for an optimum or near optimum point.

The application of the methodology developed in this thesis to the problems of optimizing test length (if the uncertainties involved in the prediction of test cost and modification cost can be overcome) should prove to be a valuable addition to the instruments of Life Cycle Cost Analysis.

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APPENDIXES

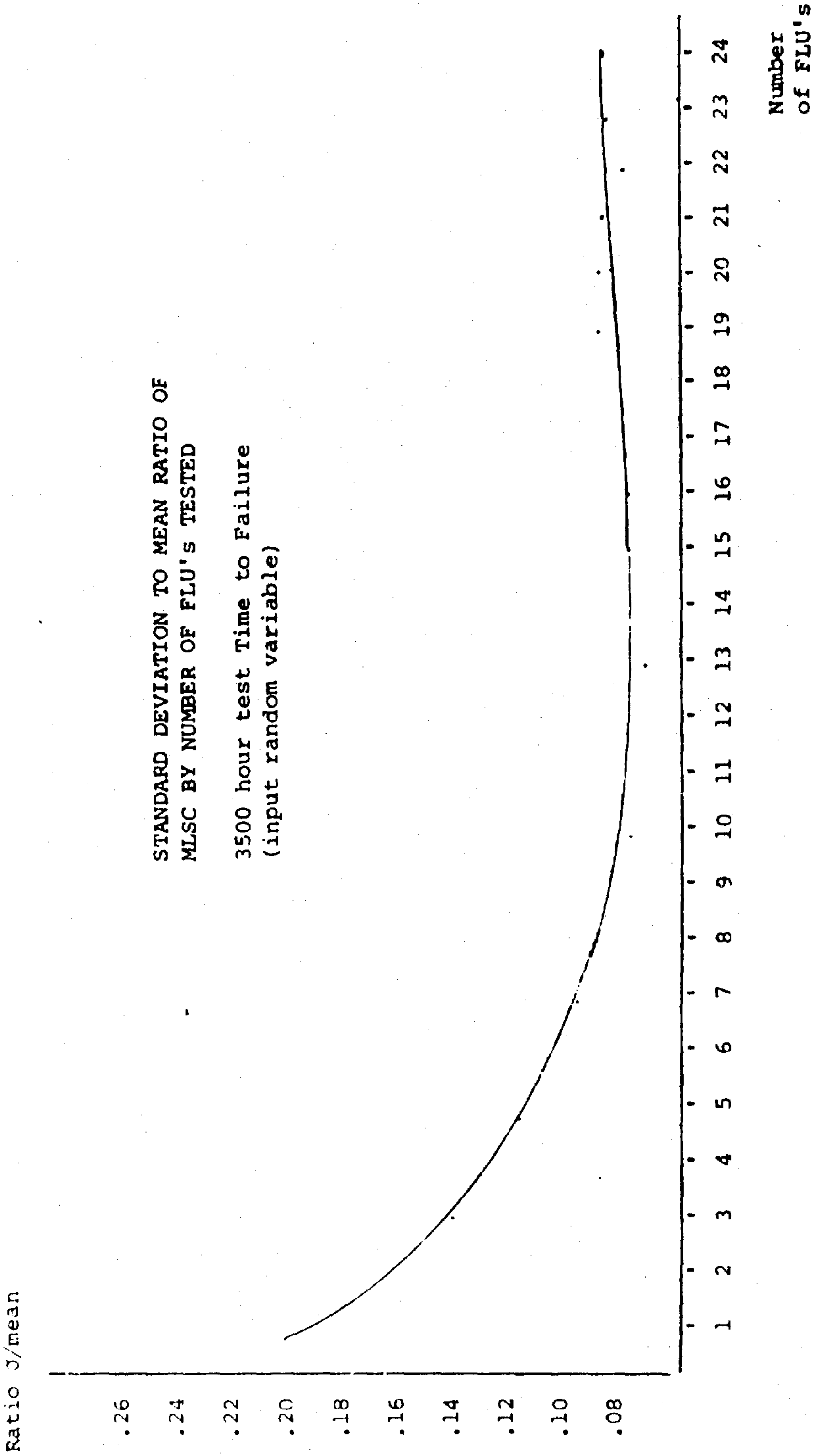
APPENDIX A

ANALYSIS OF STANDARD DEVIATION TO MEAN RATIO OF MLSC

This analysis was accomplished using the Control FLU's in the order in which they are listed in Table II So; for one FLU, the Headup Display was used, for two

FLU's the Headup Display and the Navigation Unit were used, etc.

FLU's 1 through 8 were the aircraft FLU's. FLU's 8 through 12 were the Westinghouse radar FLU's. Beginning with FLU number 13, the Headup Display and other aircraft FLU's were introduced for a second time, with FLU's number 21 through 25 being the Hughes radar FLU's. It appears that there may be a minimum ratio in the vicinity of 11 to 13 FLU's. Further analysis is nexessary to determine whether this minimum can be expected in all applications or whether it may be a result of the repeated use of the eight aircraft FLU's or other anomalies of the particular data in this application.


STANDARD DEVIATION TO MEAN RATIO OF
MLSC BY NUMBER OF FLU's TESTED
3500 hour test Time to Failure
(input random variable)
Ratio σ/mean
.26
.24
.22
.20
.18
.16
.14
.12
.10
.08
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
Number
of FLU's

APPENDIX B

DETAILED SPECIFICATIONS FOR

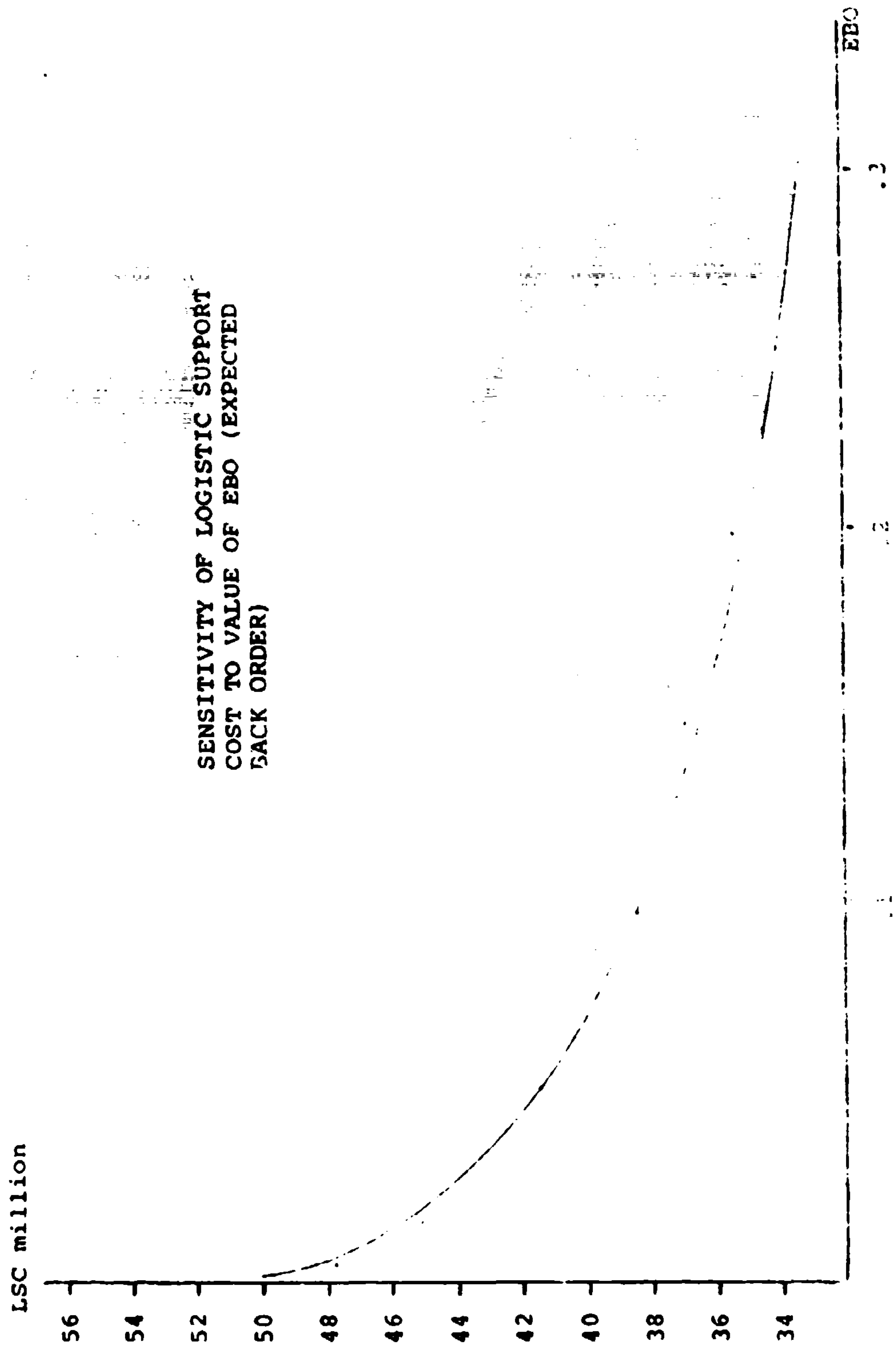
CONTROL FLU's

FLU	MTBF	UC	UC	RIP	RTS	PAMH	HIMH	RMH	BMH	DMH
Headup Display	172	42283	1.	0	.95	0	0	1.0	4.0	6.0
Navigation Unit	200	116546	1.	0	.95	.1	0	1.0	6.0	12.
Fire Control Computer	428	69106	1.	0	.95	.1	0	1.0	6.0	8.0
HUD Electronics	285	31408	1.	0	.95	.1	0	1.0	4.0	6.0
Flight Control Computer	144	50332	1.	.01	.98	.6	1.	1.	6.0	12.
Radar EO Display	188	61840	1.	0	.95	0	0	1.0	4.0	9.0
Digital Scan Converter	274	68024	1.	0	.95	.1	0	1.0	6.0	8.0
EO Display Electronics	188	7659	1.	0	.95	.1	0	1.0	4.0	[illegible]
WESTINGHOUSE RADAR FLU's										
Antenna Servo	[illegible]	68529	2	0	.9	0	0	.5	3.0	8.0
Low Power RF	338	84045	2	0	.85	0	0	.25	5.0	[illegible]
Digital Processor	150	181020	2	0	.98	0	0	.25	2.0	4.4
Trans-mitter	338	102147	2	0	.3	0	0	.67	3.0	8.0
HUGHES RADAR FLU's										
Receiver Exciter	340	46289	1.3	0	.97	0	0	.33	1.77	5.0
Data Processor	274	76158	1.3	0	.99	0	0	.25	1.2	5.0

FLU	MTBF	UC	UF	RIP	RTS	PAMH	HIMH	RMH	BMH	DMH
Signal Processor	282	89346	1.3	0	.99	0	0	.25	1.1	5.0
Transmitter	194	50298	1.3	0	.92	0	0	.53	2.04	8.0
Antenna	315	45902	1.3	0	.56	0	0	.83	1.6	6.0

APPENDIX C

SENSITIVITY ANALYSIS OF LSC TO THE VALUE OF EBO


SENSITIVITY OF LOGISTIC SUPPORT COST TO VALUE OF EBO (EXPECTED BACK ORDER)
LSC million
56
54
52
50
48
46
44
42
40
38
36
34
EBO
.1
.2
.3

SIMULATION MODEL, VARIABLE NAMES

BMH - base manhours, contract specified, same as MMOA model

BMHM - measured base manhours

DMH - depot manhours, contract specified, same as MMOA model

DMHM - measured depot manhours

HIMH - in place manhours, contract specified, same as IMH in MMOA model

PAMH - preparation and access manhours, contract specified same as MMOA model

PAMHM - measured depot manhours

RMH - remove and replace manhours, contract specified, same as MMOA model

RMHM - measured remove and replace manhours

RTS - reparable this station, contract specified, same as MMOA model

RTSM - measured fraction reparable this station

RIP - reparable in place, contract specified, same as MMOA model

TBF - contract specified MTBF, same as MTBF in MMOA model

TBFM - measured MTBF

RLSC - measured Logistic Support Cost

The Computer Program and Random Number Generation

The computer program which is included in this appendix is a complete listing of the model used in this study. The program has been written using the FORTRAN IV programming language. The model was designed for use in an interacting time sharing mode, as evidenced by the series of questions requiring user response to initiate the program. When operated in a time sharing mode on a General Electric/ Honeywell 600 series computer system, both the input and the output are at the same remote terminal.

The FORTRAN programming is straightforward and should be clear to those familiar with the language. One point, however, which may cause some confusion is the inclusion of two random number generators. The first random number generator, RAND(R), is the primary generator and the one which was used in this entire study. The second generator RND(Y), was included as an auxiliary routine in case the capacity of the first generator was exceeded.

Since approximately 2,600,000 random number generations are required on the average, (For three random input variables), it can be seen that a random number generator with a short period would eventually repeat itself in this model, thereby invalidating the results. To guard against an undetected occurrence of repetition, the first 20 sorted values of MLSC are printed out for every simulation run. Exact duplications of the same MLSC are symptomatic of repetition in the random number generator. Experience

with periodicity in this model has shown that it causes noticeable duplication in MLSC's and a reduction in the variance of the distribution.

The primary generator for this routine, RAND(R), is of the multiplicative congruential type. It utilizes the following recursive relationship:

$$n_{i+1} = a \cdot n_i \pmod{p^e}$$

where p is the number of numerals in the computer number system, 2 in this case, and e is the number of digits in a word, in this case 35.

For this generator to develop its maximal period, a must be relatively prime to $m(m=p^e)$, and a must be an odd integer. Finally, a must satisfy the following;

$$a = 8t \pm 3$$

where t is any positive integer (33:236).

For the generator used in this program:

$$t = \frac{1200703125 + 3}{8} = 152587891$$

So that the above recursive relationship is satisfied.

The maximal period is also dependent upon the starting value, n_0, called the seed. This seed must satisfy two conditions:

1. n_0 must be an odd integer
2. $n_0 = 8t+3$, where t is a positive integer

In this simulation model, two seeds were used: 317 and 11. Both satisfy the above relationship.

$$t = \frac{317 + 3}{8} = 40 \qquad t = \frac{11 - 3}{8} = 1$$

It has been shown then, that the conditions for a maximal period have been satisfied (33:236).

Derivation of Exponential Lifetimes from Constant Failure Rate Assumption

As stated in Chapter 4, the assumption of an exponential distribution of lifetimes derives inevitably from one assumption regarding the physical nature of the equipment. That assumption is that the equipment has a constant failure rate over time. For this kind of equipment, chance alone dictates when a failure will occur and the chronological age of the equipment does not affect the issue. The assumption of no deterioration over time, or a constant failure rate is commonly made for electronic equipment (5). Given this assumption then:

First: Define Reliability $R(t) = 1-F(t)$ where $F(t)$ is the CDF of the appropriate distribution of failures. $R(t)$ is the probability of survival at time t.

Define the Hazard rate $Z(t)$ as the conditional failure rate function: $Z(t) = \frac{f(t)}{R(t)}$

where $f(t)$ is the PDF of the appropriate distribution of failures. The probability of survival from time t to time $t+\Delta t$, given survival until time t, is $Z(t)\Delta t$.

And since $f(t)$ is $\frac{dF(t)}{dt}$

and $R(t) = 1-F(t)$

then $f(t) = -\frac{dR(t)}{dt}$

so that $Z(t) = -\frac{dR(t)/dt}{R(t)}$

Or $Z(t) = \frac{-d\text{Ln}R(t)}{dt}$

and $\text{Ln}R(t) = -\int_0^t Z(t)dt$

so $R(t) = e^{-\int_0^t Z(t)dt}$

Now suppose an equipment has a constant failure rate or equivalently a constant Hazard rate.

Then $Z(t) = \lambda$

and $R(t) = e^{-\int_0^t \lambda dt}$

$= e^{-\lambda t}$

$F(t) = 1-e^{-\lambda t}$

$\frac{dF(t)}{dt} = f(t) = \lambda e^{-\lambda t}$

And f(t) above is just the probability density function for the exponential distribution (12:323).

Derivation of Monte Carlo Process Generators

To develop a process generator for computer simulation, it is necessary to find what is often called the inverse transformation of the cumulative density function. In the case of the exponential density function, the CDF is:

$F(t) = 1-e^{-\lambda t}$

Solving this equation for t gives the inverse transformation

$t = \frac{-1}{\lambda} \text{Ln}(1-F(t))$

Now, if F(t) is a random fraction uniformly distributed

on the interval (0,1) then t will be a random exponential variate.

The exponential process generator, then is simply:

$$t = \frac{-1}{\lambda} \operatorname{Ln}(1-F(t))$$

Or using the same symbology as the simulation program:

$$ALIFI = \frac{-1 \operatorname{Ln}(1.-YFL)}{BETA1}$$

Where: BETA1 is the contract MTBF specification and YFL is a random fraction.

The Weibull process generator is found in an exactly parallel fashion. The weibull CDF is:

$$F(t) = 1-e^{\left(-\left(\frac{t}{\theta_1}\right)^{\theta_2}\right)}$$

The Weibull location parameter here will always be assumed to be zero.

Solving for t: $-\left(\frac{t}{\theta_1}\right)^{\theta_2} = \operatorname{Ln}(1-F(t))$

$$t = \theta_1 (\operatorname{Ln}(1-F(t)))^{1/\theta_2}$$

Since F(t) is a random fraction, (1-F(t)) is also a random fraction, so that F(t) may be substituted for (1-F(t)), and:

$$t = \theta_1 (\operatorname{Ln}(F(t)))^{1/\theta_2}$$

θ_2 is the shape parameter

θ_1 can be found by the following for Weibull failures:

$$E(t) = MTBF$$

And for Weibull variates, $E(t) = \theta_1 \Gamma(1/\theta_2 + 1)$

So $$\theta_1 = \frac{MTBF}{\Gamma(1/\theta_2 + 1)}$$

And $$t = \frac{MTBF}{\Gamma(1/\theta_2 + 1)} (-Ln(F(t)))^{1/\theta_2}$$

Using the symbology of the simulation program for Weibull failures:

$$ALIF1 - \frac{TBF\ (-Ln(YFL))^{1/\theta_2}}{\Gamma(1/\theta_2 + 1)}$$

Or for Weibull manhour generators:

$$AJOB = \frac{PAMH\ (-Ln(YFL))^{1/\theta_2}}{(1/\theta_2 + 1)}$$

The Lognormal generator is found by first deriving a normal process generator, and then raising e to the power of the normal variates. Since the cumulative density function of the normal distribution cannot be solved for t, an approximate relationship is presented here without proof (27:260).

The generator: $V = (-2Ln(YFL1))^{1/2}\cos(2\pi YFL2)$ where YFL1 and YFL2 are independent random fractions, will produce normal variates of mean 0, and standard deviation 1.0. To convert these variates of mean TMU, and standard deviation SD:

$$V1 = V\cdot(SD) + TMU$$

And finally, to convert this to a Lognormal generator:

$$V2 = e^{V1}$$

The symbology of the computer simulation is identical to that above, except that:

$$BMH1 = 2.718^{V1}$$

The Kolmogorov-Smirnov Test
For Normality of MLSC Distribution
with Mean 38.4 and Standard Deviation 2.8

t	S(t)	F(t)	|D|
29.88	.002	.0015	.0005
30.77	.005	.0043	.0007
31.68	.012	.0104	.0016
32.59	.018	.0244	.0064
33.49	.050	.0505	.0005
34.40	.0860	.0934	.0074
35.31	.152	.1611	.0091
36.21	.246	.2546	.0086
37.12	.362	.3707	.0087
38.03	.494	.50	.006
38.94	.618	.6255	.0075
39.84	.732	.7389	.0069
40.75	.822	.8315	.0095
41.66	.873	.898	.025
42.56	.924	.9452	.0212
43.47	.952	.9719	.0199
44.38	.971	.9871	.0161
45.29	.908	.9946	.0146
46.19	.983	.9979	0.0

S(t) is the empirical function

F(t) is the theoretical function

D is the Kolmogorov-Smirnov statistic

For $\alpha = .10$, $D_\alpha = .0836$

The null hypothesis is:

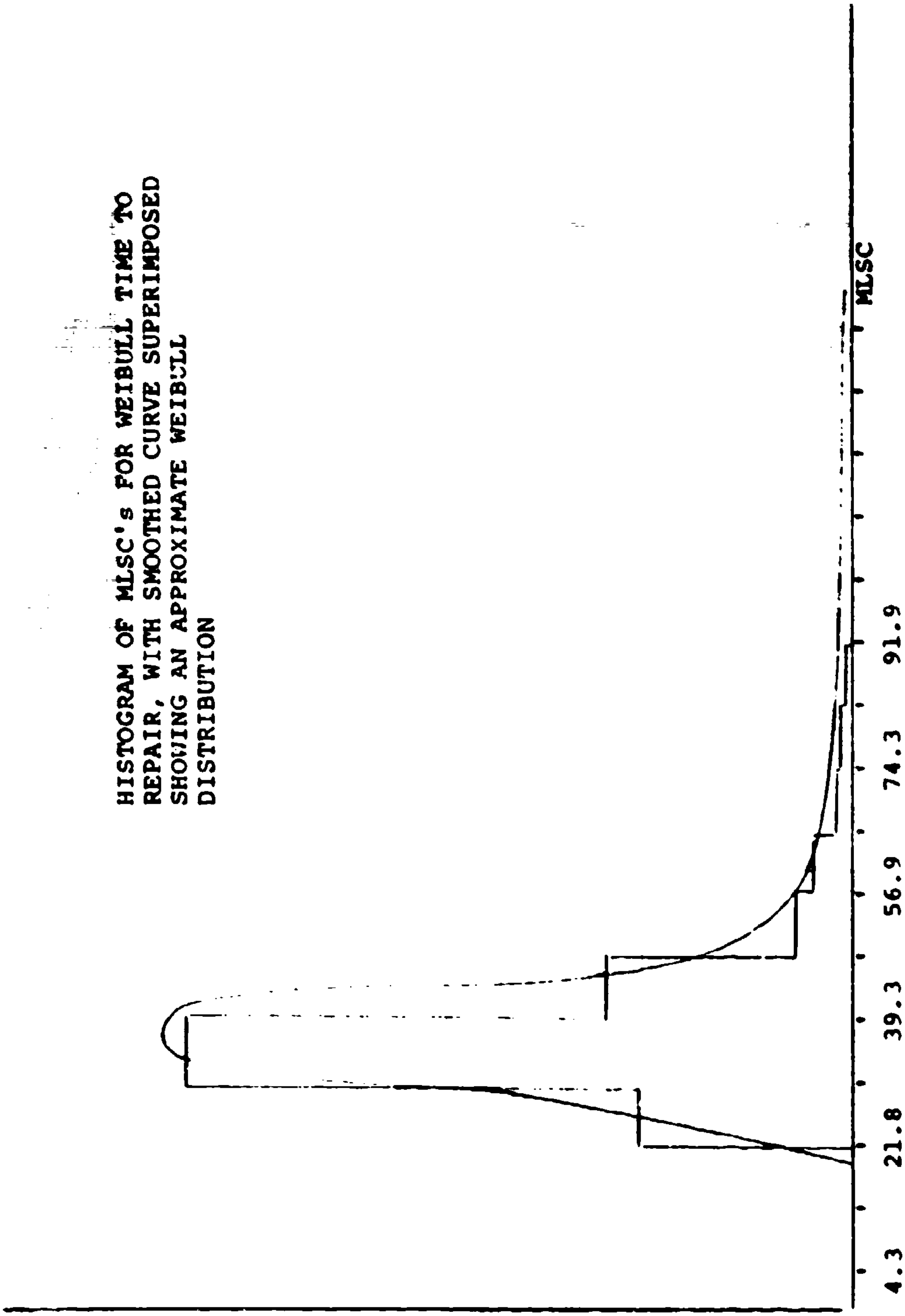
H_0: The distribution is Normal with parameters 38.4, 2.8

The alternate hypothesis is:

H_a: The distribution is not Normal with parameters 38.4, 2.8

Decision rule: If $|D| \geq D_\alpha$ Reject H_0

Since $D<D_\alpha$ For all values of t, do not reject H_0



HISTOGRAM OF MLSC's FOR WEIBULL TIME TO REPAIR, WITH SMOOTHED CURVE SUPERIMPOSED SHOWING AN APPROXIMATE WEIBULL DISTRIBUTION

The Kolmogorov-Smirnov Test for Weibull Distribution of MLSC's with Parameters θ_1=43.2 and θ_2=2.8

t	S(t)	F(t)	|D|
30.6	.171	.318	.147
39.33	.703	.539	.164
48.0	.899	.741	.157
56.9	.943	.88	.063
65.6	.973	.96	.013
74.3	.981	.99	.009
83.1	.988	.998	.010
.	.	.	.
.	.	.	.
.	.	.	.

All remaining values of D are less than .01

S(t) is the empirical function

F(t) is the theoretical function

D is the Kolmogorov-Smirov statistic

For α=.10, D_α=.0836

The null hypothesis is:

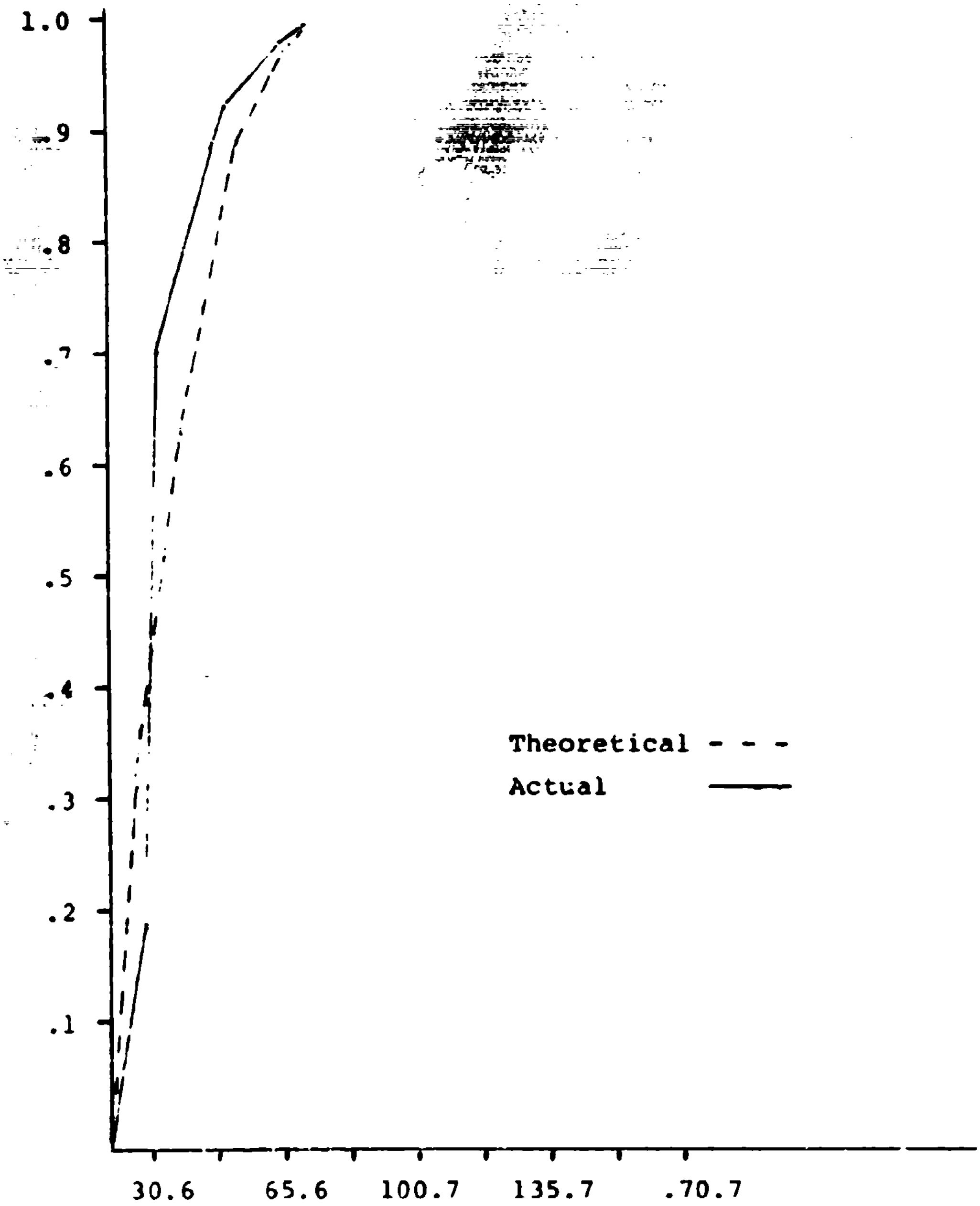
H_0: The distribution is Weibull with parameters 43.2, 2.8

The alternate hypothesis is:

H_a: The distribution is not Weibull with parameters 43.2, 2.8

Decision rule: If $|D| \geq D_\alpha$ Reject H_0

Since $|D| > D_\alpha$ In several instances, the null hypothesis is rejected. Notice that this only says that the distribution is not Weibull (43.2, 2.8). No inference is made as to what the real distribution is.



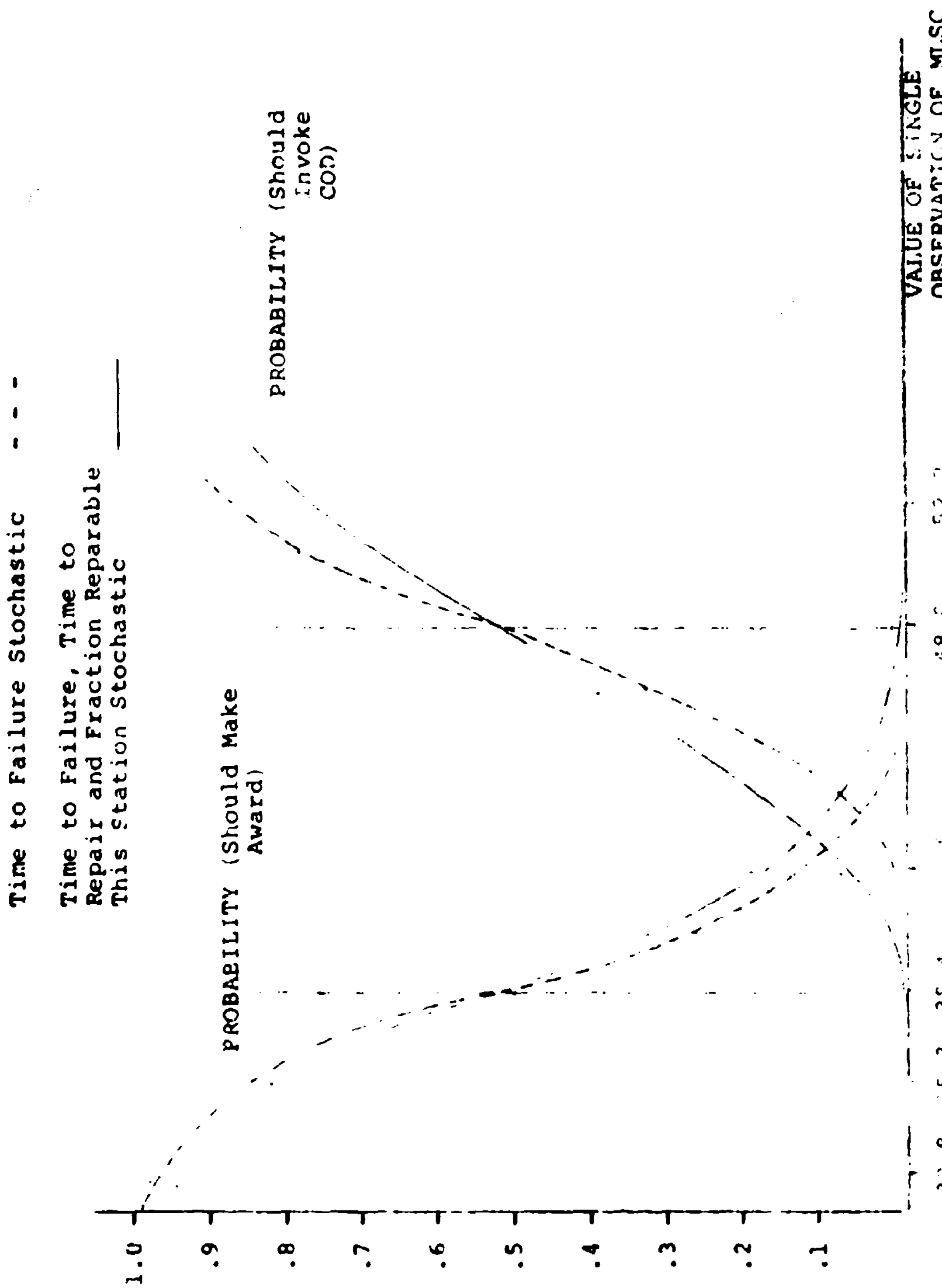
Kolmogorov-Smirnov Test For Weibull Distrubition

APPENDIX E

DECISION CURVES

APPENDIX E

DECISION CURVES


Time to Failure Stochastic
Time to Failure, Time to Repair and Fraction Reparable This Station Stochastic
PROBABILITY (Should Make Award)
PROBABILITY (Should Invoke COD)
VALUE OF SINGLE OBSERVATION OF MLSC
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
32.8
48.0

APPENDIX F

AFLC LSC MODEL VARIABLE NAMES

FLU VARIABLES

1. BMC — Average cost per FLU repaired at base level for stockage and repair of lower level assemblies expressed as a fraction of the FLU unit cost. This is the implicit repair disposition cost for a FLU representing labor, material and stockage for lower indenture components within the FLU. (e.g. shop reparable units or modules).

2. BMH — Average manhours to perform intermediate level (base shop) maintenance on a removed FLU including fault isolation, repair and verification.

3. BRCT — Average base repair cycle time.

4. COND — Fraction of removals expected to result in condemnation at base level.

5. DMC — Same as BMC except refers to depot repair actions.

6. DMH — Same as BMH except refers to depot level maintenance.

7. DRCT — Average depot repair cycle time.

8. IMH — Average manhours to perform corrective maintenance of the FLU in place or on line, including fault isolation, repair and verification

9. K — Number of line items of peculiar support equipment used in repair of the FLU.

10. MTBF — Mean time between failures in operating hours of the FLU in the operational environment.

11. NRTS — Fraction of removals expected to be returned to depot for repair.

12. PA — Number of new "P" coded reparable assemblies within the FLU.

13. PAMH — Average manhours expended for preparation and access for the FLU. For example, jacking, unbuttoning, removal of other units and hookup of support equipment.

14. PP Number of new "P" coded consumable items within the FLU.

15. QPA Quantity of like FLU's within the system. (Quantity per Application).

16. RIP Fraction of FLU failures which can be repaired in place or on line.

17. RMH Average manhours to fault isolate, remove and replace the FLU and verify restoration of the system to operational status.

18. RTS Fraction of removals expected to be repaired at base level.

19. SP Number of Standard (already stock listed) parts within the FLU.

20. UC Expected unit cost of the FLU at the time of initial provisioning.

21. UF Ratio of operating hours per flying hour for the FLU.

22. W FLU weight in pounds.

PECULIAR SUPPORT EQUIPMENT DATA

1. CAB Cost per unit of support equipment for base level.

2. CAD Cost per unit of support equipment for depot.

3. DOWN Fraction of total available operating time that the unit of support equipment will be down for maintenance and calibration.

SYSTEM DATA

1. BA Cost of additional common shop support equipment for the system.

2. BAA Available work time in the base shop in manhours per month.

3. BLR Base labor rate.

4. BMR Base consumable material consumption rate.

5. DA Cost of additional common depot support equipment for the system.

6. DAA Available work time at the depot in manhours per month.

7. DLR Depot labor rate.

8. DMR Depot consumable material consumption rate.

9. FLA Cost of additional common or peculiar flight line support.

10. N Number of different FLUs within the system.

11. PPHM Average manhours to perform a preflight and a post-flight inspection on back-to-back sorties.

12. SLLRMH Average manhours to perform Service, Load, Launch, and Recovery for one sortie including fueling, lubrication, munitions loading, etc.

13. SMH Average manhours to perform a scheduled periodic or phased inspection on the system.

14. SMI Flying hour interval between scheduled (periodic or phased) inspection.

15. TFMH Average manhours to perform a through-flight inspection between back-to-back sorties.

PROPULSION SYSTEM DATA

1. ARBUT Engine Automatic Resupply and Buildup Time in months.

2. BP Base engine repair cycle time in months.

3. CMRI Average engine operating hours between removals.

4. CONF Probability of satisfying a demand for a whole engine.

5. DP Depot Engine Repair Cycle Time in months.

6. EPA Number of engines per aircraft.

7. ERMH — Average manhours to remove and replace a whole engine including engine trim and runup time.

8. ERTS — Fraction of engine removals expected to be repaired at base level. (0 < ERTS < 1).

9. EUC — Expected unit cost of a whole engine.

10. FC — Fuel cost per unit.

11. FR — Fuel consumption rate in gallons per flying hour. This rate is an average value.

12. LS — Number of stockage locations for spare engines.

WEAPON SYSTEM DATA

1. BUR — Support Equipment Utilization Rate--base level.

2. COB — Annual cost to operate and maintain a unit of support equipment at base level expressed as a fraction of the unit cost (CAB).

3. COD — Same as COB except refers to depot support equipment.

4. DUR — Support Equipment Utilization Rate--depot level

5. EBO — Standard established for expected backorders.

6. M — Number of operating base locations.

7. MRO — Average manhours per maintenance action for completing on-equipment maintenance records.

8. MRF — Average manhours per maintenance action for completing off-equipment maintenance records.

9. NSUB — Number of systems.

10. OS — Fraction of total force deployed to overseas locations.

11. OSTCON — Average order and shipping within the CONUS.

12. OSTOS — Average order and shipping time to overseas locations.

13. PFFH — Expected peak force flying hours per month.

14. PIUP — Operational service life of the weapon system in years. (Program Inventory Usage Period).

15. PMB Direct productive manhours per man year at the base level.

16. PMD Same as PMB for depot level.

17. PSC Packing and shipping costs in dollars per pound for CONUS.

18. PSO Same as PSC for overseas.

19. RAC Recurring inventory management cost for reparable assembly in the wholesale system.

20. SA Annual base supply inventory management cost.

21. SFH Average sortie length in flying hours.

22. SR Average manhours maintenance action for completing supply transaction records.

23. TD Cost per original page of technical documentation.

24. TF Fraction of total sorties which are flown back-to-back.

25. TFFH Expected total force flying hours over the Program Inventory Usage Period.

26. TR Average manhours per maintenance action for completing transportation records.

27. TRB Annual turnover rate for base personnel.

28. TRD Annual turnover rate for depot personnel.

VITA

The author received his bachelor's degree in Basic Science from the United Stated Air Force Academy in 1965. Following pilot training, he gained his combat experience flying F4C/D's at Danang Air Base Republic of Vietnam in 1967-1968.

During the period of 1968-1971, the author flew operational test and evaluation missions for the Tactical Air Warfare Center, evaluating air to air and air to ground munitions and electronic countermeasures in F4D/E and RF4/C aircraft.

The author served as an instructor pilot, flight examiner and Assistant Chief of Standardization and Evaluation with the 355 Tactical Fighter Wing flying A7D's from 1971 through 1974.

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